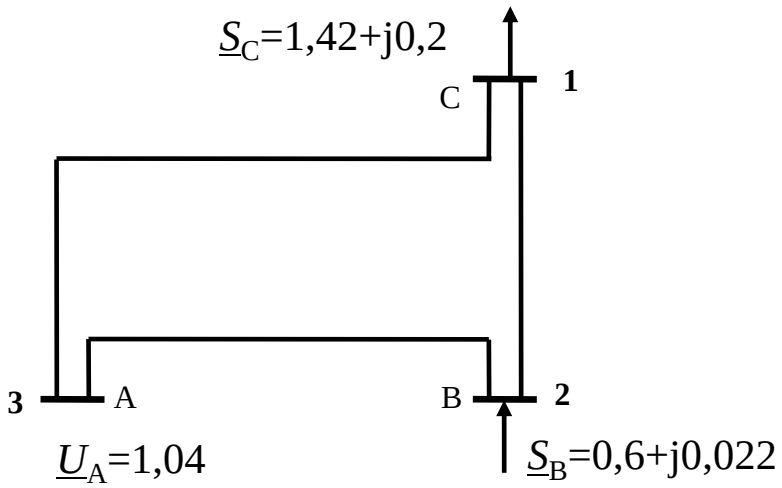
Пример 4.1 Систем без PU



Подредување на јазлите:

- PQ јазли
- PU јазли
- Јазол со познат напон (S)

За Гаус-Зајделовиот метод подредувањето не е неопходно, но во овој случај тоа е направено заради споредба со решенијата добиени со другите методи

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,04	0	S	3
C	–1,42	–0,2	?	?	PQ	1
B	0,6	0,022	?	?	PQ	2

Податоци за гранките (per unit)

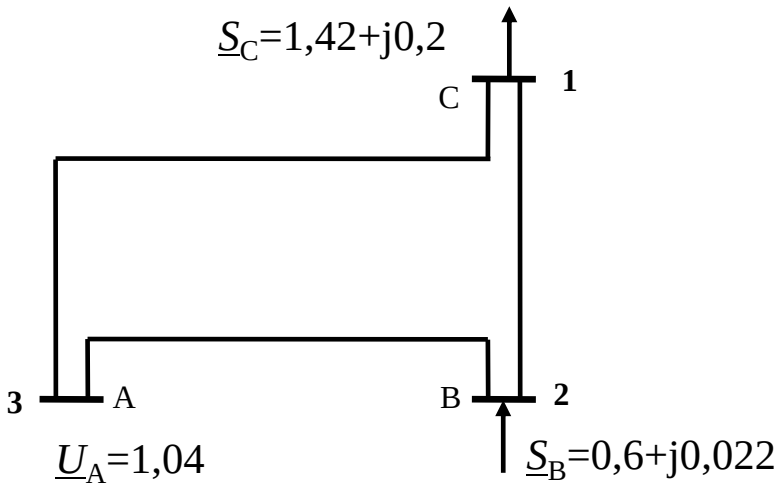
Гранка	R	X	G'	B'
B-C	0,24	0,32	0	0,04
A-C	0,12	0,16	0	0,02
A-B	0,24	0,32	0	0,04

$\underline{U}_C^{(0)} = 1{,}0 + j0{,}0 \quad \underline{U}_A = 1{,}04 \quad \underline{U}_B^{(0)} = 1{,}0 + j0{,}0$

Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0,24	0,32	0	0,04
A-C	1-3	0,12	0,16	0	0,02
A-B	2-3	0,24	0,32	0	0,04

Пример 4.1 Систем без PU



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0,24	0,32	0	0,04
A-C	1-3	0,12	0,16	0	0,02
A-B	2-3	0,24	0,32	0	0,04

$$\underline{Y}_{A-B} = \frac{1}{\underline{Z}_{A-B}} = \frac{1}{0{,}24 + j0{,}32} = 1{,}5 - j2$$

$$\underline{Y}'_{A-B} = 0 + j0{,}04$$

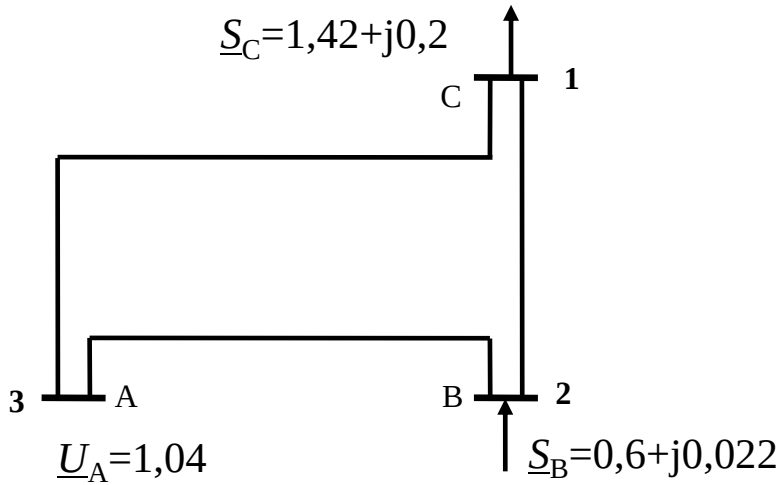
$$\underline{Y}_{B-C} = \frac{1}{\underline{Z}_{B-C}} = \frac{1}{0{,}24 + j0{,}32} = 1{,}5 - j2$$

$$\underline{Y}'_{B-C} = 0 + j0{,}04$$

$$\underline{Y}_{C-A} = \frac{1}{\underline{Z}_{C-A}} = \frac{1}{0{,}12 + j0{,}16} = 3 - j4$$

$$\underline{Y}'_{C-A} = 0 + j0{,}02$$

Пример 4.1 Систем без PU



$$\underline{Y}_{A-B} = 1{,}5 - j2 \qquad \underline{Y}'_{A-B} = 0 + j0{,}04$$

$$\underline{Y}_{B-C} = 1{,}5 - j2 \qquad \underline{Y}'_{B-C} = 0 + j0{,}04$$

$$\underline{Y}_{C-A} = 3 - j4 \qquad \underline{Y}'_{C-A} = 0 + j0{,}02$$

$$\underline{Y}_{AA} = \underline{Y}_{C-A} + \underline{Y}_{A-B} + \frac{1}{2}\cdot\left(\underline{Y}'_{C-A} + \underline{Y}'_{A-B}\right) = 4{,}5 - j5{,}97$$

$$\underline{Y}_{BB} = \underline{Y}_{B-C} + \underline{Y}_{A-B} + \frac{1}{2}\cdot\left(\underline{Y}'_{B-C} + \underline{Y}'_{A-B}\right)$$

$$= 1{,}5 - j2 + 1{,}5 - j2 + \frac{1}{2}\cdot(j0{,}04 + j0{,}04) = 3 - j3{,}96$$

$$\underline{Y}_{CC} = \underline{Y}_{C-A} + \underline{Y}_{B-C} + \frac{1}{2}\cdot\left(\underline{Y}'_{C-A} + \underline{Y}'_{B-C}\right)$$

$$= 3 - j4 + 1{,}5 - j2 + \frac{1}{2}\cdot(j0{,}04 + j0{,}02) = 4{,}5 - j5{,}97$$

$$\underline{Y}_{CA} = \underline{Y}_{AC} = -\underline{Y}_{C-A} = -(3 - j4) = -3 + j4$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{B-C} = -(1{,}5 - j2) = -1{,}5 + j2$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = -\underline{Y}_{A-B} = -(1{,}5 - j2) = -1{,}5 + j2$$

$$\underline{\mathbf{Y}} = \begin{matrix} \\ C \\ B \\ A \end{matrix}\begin{matrix} \begin{matrix} C & B & A \end{matrix} \\ \begin{bmatrix} 4{,}5 & -1{,}5 & -3{,}0 \\ -1{,}5 & 3{,}0 & -1{,}5 \\ -3{,}0 & -1{,}5 & 4{,}5 \end{bmatrix} \end{matrix} + j\begin{matrix} \begin{matrix} C & B & A \end{matrix} \\ \begin{bmatrix} -5{,}97 & 2{,}00 & 4{,}00 \\ 2{,}00 & -3{,}96 & 2{,}00 \\ 4{,}00 & 2{,}00 & -5{,}97 \end{bmatrix} \end{matrix}\begin{matrix} \\ C \\ B \\ A \end{matrix}$$

Пример 4.1 Систем без PU

$$\underline{U}_{\mathrm{C}}^{(1)} = \frac{1}{\underline{Y}_{\mathrm{CC}}} \cdot \left[\frac{P_{\mathrm{C}} - \mathrm{j}Q_{\mathrm{C}}}{\left(\underline{U}_{\mathrm{C}}^{(0)}\right)^{*}} - \underline{Y}_{\mathrm{CB}}\underline{U}_{\mathrm{B}}^{(0)} - \underline{Y}_{\mathrm{CA}}\underline{U}_{\mathrm{A}} \right]$$

$$= \frac{\left[\frac{-1,42 - \mathrm{j}(-0,2)}{(1+\mathrm{j}0)^{*}} - (-1,5+\mathrm{j}2)\cdot(1+\mathrm{j}0) - (-3+\mathrm{j}4)\cdot(1,04+\mathrm{j}0) \right]}{4,5 - \mathrm{j}5,97}$$

$$= \frac{3,2 - \mathrm{j}5,96}{4,5 - \mathrm{j}5,97} = (0,8942636 - \mathrm{j}0,1380547)$$

$$\underline{U}_{\mathrm{B}}^{(1)} = \frac{1}{\underline{Y}_{\mathrm{BB}}} \cdot \left[\frac{P_{\mathrm{B}} - \mathrm{j}Q_{\mathrm{B}}}{\left(\underline{U}_{\mathrm{B}}^{(0)}\right)^{*}} - \underline{Y}_{\mathrm{BC}}\underline{U}_{\mathrm{C}}^{(1)} - \underline{Y}_{\mathrm{BA}}\underline{U}_{\mathrm{A}} \right]$$

$$= \frac{\left[\frac{0,6 - \mathrm{j}(0,022)}{(1+\mathrm{j}0)^{*}} - (-1,5+\mathrm{j}2)\cdot(0,8942636 - \mathrm{j}0,1380547) - (-1,5+\mathrm{j}2)\cdot(1,04+\mathrm{j}0) \right]}{3 - \mathrm{j}3,96}$$

$$= \frac{0,6 - \mathrm{j}0,022 + 1,065286 - \mathrm{j}1,995609 + 1,56 - \mathrm{j}2,08}{3 - \mathrm{j}3,96}$$

$$= \frac{3,225286 - \mathrm{j}4,097610}{3 - \mathrm{j}3,96} = (1,049462 + \mathrm{j}0,01941949)$$

Пример 4.1 Систем без PU

$$\Delta\underline{U}_{\mathrm{C}}^{(1)} = \underline{U}_{\mathrm{C}}^{(1)} - \underline{U}_{\mathrm{C}}^{(0)} = (0{,}8942636 - \mathrm{j}0{,}1380547) - (1 + \mathrm{j}0)$$
$$= (-0{,}1057364 - \mathrm{j}0{,}1380547)$$

$$\Delta\underline{U}_{\mathrm{B}}^{(1)} = \underline{U}_{\mathrm{B}}^{(1)} - \underline{U}_{\mathrm{B}}^{(0)} = (1{,}049462 + \mathrm{j}0{,}01941949) - (1 + \mathrm{j}0)$$
$$= (-0{,}049462 - \mathrm{j}0{,}01941949)$$

$$\varepsilon = 10^{-5} \Rightarrow \left|\Delta\underline{U}_{\mathrm{C}}^{(1)}\right| > \varepsilon \text{ i } \left|\Delta\underline{U}_{\mathrm{B}}^{(1)}\right| > \varepsilon$$

$$\underline{U}_{\mathrm{C}}^{(2)} = \frac{1}{\underline{Y}_{\mathrm{CC}}} \cdot \left[\frac{P_{\mathrm{C}} - \mathrm{j}Q_{\mathrm{C}}}{\left(\underline{U}_{\mathrm{C}}^{(1)}\right)^{*}} - \underline{Y}_{\mathrm{CB}}\underline{U}_{\mathrm{B}}^{(1)} - \underline{Y}_{\mathrm{CA}}\underline{U}_{\mathrm{A}} \right] \qquad \underline{U}_{\mathrm{B}}^{(2)} = \frac{1}{\underline{Y}_{\mathrm{BB}}} \cdot \left[\frac{P_{\mathrm{B}} - \mathrm{j}Q_{\mathrm{B}}}{\left(\underline{U}_{\mathrm{B}}^{(1)}\right)^{*}} - \underline{Y}_{\mathrm{BC}}\underline{U}_{\mathrm{C}}^{(2)} - \underline{Y}_{\mathrm{BA}}\underline{U}_{\mathrm{A}} \right]$$

„Точно“ решение после 10 или 11 итерации:

$$\underline{U}_{\mathrm{C}}^{(10)} = (0{,}8637032 - \mathrm{j}0{,}1255576) = 0{,}87 \cdot \mathrm{e}^{-\mathrm{j}8{,}27^{\circ}}$$

$$\underline{U}_{\mathrm{B}}^{(10)} = (1{,}029677 + \mathrm{j}0{,}02482077) = 1{,}03 \cdot \mathrm{e}^{\mathrm{j}1{,}38^{\circ}}$$

$$\max_{i} \left\{ \left| \underline{U}_{i}^{(\nu+1)} - \underline{U}_{i}^{(\nu)} \right| \right\} \le \varepsilon$$

Пример 4.1 Систем без PU

„Точно“ решение со MATLAB

```
>> GS
primer.4.01; Sbaz=1 MVA; Ubaz=1 kV; eps=1e-005
Y =
   4.5000 - 5.9700i  -1.5000 + 2.0000i  -3.0000 + 4.0000i
  -1.5000 + 2.0000i   3.0000 - 3.9600i  -1.5000 + 2.0000i
  -3.0000 + 4.0000i  -1.5000 + 2.0000i   4.5000 - 5.9700i

U= (+1.000000 j+0.000000) (+1.000000 j+0.000000) (+1.040000 j+0.000000)
S= (-1.420000 j-0.200000) (+0.600000 j+0.022000) (+0.000000 j+0.000000)

it=10; max_dU=9.96715e-006

U= (+0.863703 j-0.125558) (+1.029677 j+0.024821) (+1.040000 j+0.000000)
U= ( +0.86 j -0.13) ( +1.03 j +0.02) ( +1.04 j +0.00)
U=   0.873 exp(j-8.271)   1.030 exp(j+1.381)   1.040 exp(j+0.000)

 Szad= (-1.420000 j-0.200000) (+0.600000 j+0.022000) (+0.000000 j+0.000000)
Spres= (-1.419979 j-0.199986) (+0.599998 j+0.022000) (+1.036843 j+0.369400)
   dS= (-0.000021 j-0.000014) (+0.000002 j-0.000000)

Zagubi (+0.216843 j+0.191400)
>>
```

Пример 4.1 Систем без PU

„Точно“ решение со програм напишан во FORTRAN

```
 Vlezni podatoci za grankite
-------------------------------------------------------------------------------
           R             X        G'xE-06       B'xE-06       Prenosen odnos
-------------------------------------------------------------------------------
 A - B 0.2400000     0.3200000      0.000000      40000.00       1.00/   1.00
 B - C 0.2400000     0.3200000      0.000000      40000.00       1.00/   1.00
 A - C 0.1200000     0.1600000      0.000000      20000.00       1.00/   1.00
-------------------------------------------------------------------------------
      Vlezni podatoci za jazlite
-------------------------------------------------------------------------
Jazol     P              Q           U             Theta       Tip   Ubaz
-------------------------------------------------------------------------
   C -1.420000     -0.2000000         ?              ?           PQ   1.00
   B 0.6000000      0.2200000E-01     ?              ?           PQ   1.00
   A    ?              ?           1.040000      0.000000        S   1.00
-------------------------------------------------------------------------
 Parametri od pi-ekv. semi
 3-2    Y,Y'/2(  1.500000     j -2.000000    )(  0.000000     j 0.2000000E-01)
 2-1    Y,Y'/2(  1.500000     j -2.000000    )(  0.000000     j 0.2000000E-01)
 3-1    Y,Y'/2(  3.000000     j -4.000000    )(  0.000000     j 0.1000000E-01)
      Formiranje na matricata Y so inspekcija na mrezata
      ----------------------------------------------
           Matrica G                                  Matrica B
        C             B             A              C             B             A
 C  4.500000     -1.500000     -3.000000     C -5.970000      2.000000      4.000000
 B -1.500000      3.000000     -1.500000     B  2.000000     -3.960000      2.000000
 A -3.000000     -1.500000      4.500000     A  4.000000      2.000000     -5.970000
Resenie so Gauss-Seidel-ov metod; alfa=   1.000000     ; eps=  1.0000001E-05
```

Пример 4.1 Систем без PU

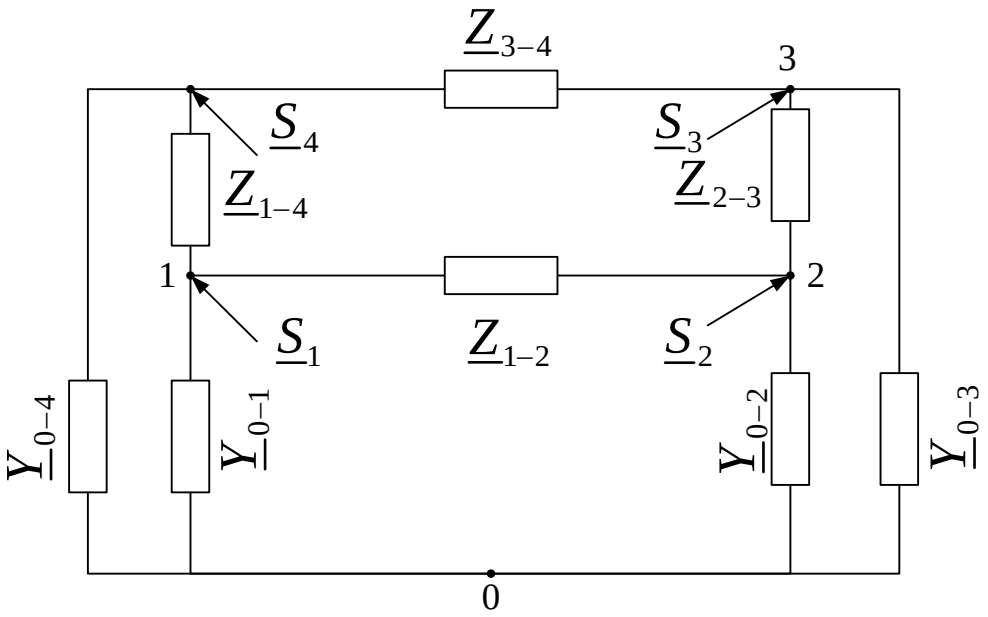
„Точно“ решение со програм напишан во FORTRAN

```
 Naponi vo jazlite posle  11.0 iteracii; (dU)max= 0.3883947E-05 < 0.1000000E-04
-------------------------------------------------------------------------------------------
   (      E       +j      F       )  (    U         [ejth (rad.)]    )  (        U[ejth]      )
-------------------------------------------------------------------------------------------
 C ( 0.8636993     j-0.1255583    )  ( 0.8727780     ej-0.1443614    )  (     0.87 ej -8.27130)
 B (  1.029675     j 0.2482079E-01)  (  1.029974     ej 0.2410079E-01)  (     1.03 ej  1.38087)
 A (  1.040000     j  0.000000    )  (  1.040000     ej  0.000000    )  (     1.04 ej  0.00000)
-------------------------------------------------------------------------------------------

                  Zadadeni i presmetani moknosti vo jazlite (p.u.)
-------------------------------------------------------------------------------------------
       (      P       +j      Q       )(      P       +j      Q       )(     dP       +j      dQ      )
-------------------------------------------------------------------------------------------
C(PQ)   ( -1.420000     j-0.2000000    )( -1.419992     j-0.1999947    )(-0.8344650E-05 j-0.5319715E-05)
B(PQ)   ( 0.6000000     j 0.2200000E-01)( 0.5999990     j 0.2199967E-01)( 0.1072884E-05 j 0.3259629E-06)
A(S )   (  0.000000     j  0.000000    )(  1.036860     j 0.3694165    )(  0.000000     j  0.000000    )
-------------------------------------------------------------------------------------------
Zagubi: ( 0.2168670     j 0.1914215    )
```

Гаус-Зајделов метод

- Пример 4.2



$$\underline{U}_4 = (112 + j0)\ \text{kV}$$

	MVA
$\underline{S}_1$	-24 - 18i
$\underline{S}_2$	-36 - 17.433i
$\underline{S}_3$	-17 - 10.5366i
$\underline{S}_4$	?

	Ω/μS
$\underline{Z}_{1\text{-}2}$	1.2+4i
$\underline{Z}_{1\text{-}4}$	2.4+8i
$\underline{Z}_{2\text{-}3}$	3.6+12i
$\underline{Z}_{3\text{-}4}$	2.4+8i
$\underline{Y}_{0\text{-}1}$	0+39i
$\underline{Y}_{0\text{-}2}$	0+52i
$\underline{Y}_{0\text{-}3}$	0+65i
$\underline{Y}_{0\text{-}4}$	0+52i

Гаус-Зајделов метод

- Пример 4.2

```
>> GS
primer.4.02; Sbaz=100 MVA; Ubaz=110 kV; eps=1e-005
Y =
  12.4885 -41.6237i  -8.3257 +27.7523i        0            -4.1628 +13.8761i
  -8.3257 +27.7523i  11.1009 -36.9968i  -2.7752 + 9.2508i        0
        0            -2.7752 + 9.2508i   6.9381 -23.1190i  -4.1628 +13.8761i
  -4.1628 +13.8761i        0            -4.1628 +13.8761i   8.3257 -27.7460i

U= (+1.000000 j+0.000000) (+1.000000 j+0.000000) (+1.000000 j+0.000000) (+1.018182 j+0.000000)
S= (-0.240000 j-0.180000) (-0.360000 j-0.174331) (-0.170000 j-0.105366) (+0.000000 j+0.000000)

it=16; max_dU=7.72643e-006

U= (+0.991141 j-0.023956) (+0.986013 j-0.029892) (+0.999323 j-0.017440) (+1.018182 j+0.000000)
U= 0.991431 exp(j-1.385) 0.986466 exp(j-1.736) 0.999475 exp(j-1.000) 1.018182 exp(j+0.000)
U= 109.057373 exp(j-1.385) 108.511217 exp(j-1.736) 109.942291 exp(j-1.000) 112.000000 exp(j+0.000)

 Szad= (-0.240000 j-0.180000) (-0.360000 j-0.174331) (-0.170000 j-0.105366) (+0.000000 j+0.000000)
Spres= (-0.239815 j-0.179916) (-0.359975 j-0.174317) (-0.170000 j-0.105365) (+0.779404 j+0.466504)
   dS= (-0.000185 j-0.000084) (-0.000025 j-0.000014) (-0.000000 j-0.000001)

Zagubi (+0.940360 j+0.680734)
>>
```

Гаус-Зајделов метод

```
clear;
load('primer.4.02.mat');
Ubaz = Unom; Us = Us / Ubaz; Zbaz = Ubaz^2 / Sbaz;
S = S / Sbaz;
Ygr = inv(Zgr / Zbaz);
Y = A'*Ygr*A;
nj = length(Y);
% naponi na jazlite vo nultata iteracija
for i = 1 : nj-1; U(i) = complex(Unom / Ubaz,0.); end; U(nj) = complex(Us,0.);
disp(sprintf('Sbaz=%d MVA; Ubaz=%d kV; eps=%g',Sbaz, Ubaz, eps));
Y
s1 = sprintf('\nU='); s2 = 'S=';
for i = 1 : nj;
 s1 = [s1, sprintf(' (%+7f j%+7f)',real(U(i)),imag(U(i)))];
 s2 = [s2, sprintf(' (%+7f j%+7f)',real(S(i)),imag(S(i)))];
end
disp(s1); disp(s2);
[it,U,max_dU] = gauss_seidel_nlr(Y,S,U,eps,nj);
Inj = complex(0.,0.); for i = 1 : nj; Inj = Inj + Y(nj,i) * U(i); end; S(nj) = U(nj) * conj(Inj);
% Inj i S(nj) - struja i moknost injektirani vo jazolot nj, soodvetno
disp(sprintf('\nit=%2d; max_dU=%g',it, max_dU));
s1 = sprintf('\nU='); s2 = 'U='; s3 = 'U='; s4 = sprintf('\nS=');
for i = 1 : nj;
  s1 = [s1, sprintf(' (%+7f j%+7f)',real(U(i)),imag(U(i)))];
  s2 = [s2, sprintf(' (%+6.2f j%+6.2f)',real(U(i) * Ubaz),imag(U(i) * Ubaz))];
  s3 = [s3, sprintf(' %6.2f exp(j%+6.3f)',abs(U(i)) * Ubaz, 180. * angle(U(i))/pi)];
  s4 = [s4, sprintf(' (%+7.3f j%+7.3f)',real(S(i) * Sbaz),imag(S(i) * Sbaz))];
end
disp(s1); disp(s2); disp(s3); disp(s4);
Zagubi = complex(0.,0.); for i = 1:nj; Zagubi = Zagubi + S(i); end %for
disp(sprintf('Zagubi (%+7f j%+7f)',real(Zagubi * Sbaz), imag(Zagubi * Sbaz)));
```

Гаус-Зајделов метод

```
function [it,U,max_dU] = gauss_seidel_nlr(Y,S,U,eps,nj)
% Gauss-Seidel za sistemi nelinearni ravenki
  it = 0;
  max_dU = 1 / eps;
  while max_dU >= eps
    it = it + 1;
    max_dU=0;
    for i = 1 : nj-1
      U1 = conj(S(i)/U(i));
      for j = 1 : i-1
        U1 = U1 - Y(i,j)*U(j);
      end %for
      for j = i+1 : nj
        U1 = U1 - Y(i,j)*U(j);
      end %for
      U1 = U1 / Y(i,i); % U1 napon vo novata iteracija vo jazolot i
      dU = U1 - U(i); % U vo relativni edinici
      max_dU = max([max_dU,abs(dU)]);
      U(i)=U(i) + dU;
    end %for
  end %while
```

Гаус-Зајделов метод за систем со јазли со контролиран напон (PU јазли)

$$\underline{U}_i^{(\nu+1)} = \frac{1}{\underline{Y}_{ii}} \cdot \left(\frac{P_i - \mathrm{j}Q_i}{\left(\underline{U}_i^{(\nu)}\right)^*} - \sum_{l=1}^{i-1} \underline{Y}_{il} \cdot \underline{U}_l^{(\nu+1)} - \sum_{l=i+1}^{n} \underline{Y}_{il} \cdot \underline{U}_l^{(\nu)} \right); \; i = 1,2,\cdots,n; \; i \neq s$$

Ако јазолот е од типот PU, не е позната инјектираната реактивна моќност Q_i

$$\underline{I}_i = \sum_{l=1}^{n} \underline{Y}_{il} \cdot \underline{U}_l$$

$$Q_i^{(\nu+1)} = \mathrm{Im}\left\{\underline{U}_i \cdot \underline{I}_i^*\right\} = \mathrm{Im}\left\{\underline{U}_i \cdot \sum_{l=1}^{n} \left(\underline{Y}_{il} \cdot \underline{U}_l\right)^*\right\} = \mathrm{Im}\left\{\underline{U}_i^{(\nu)} \cdot \sum_{l=1}^{i-1} \left(\underline{Y}_{il} \cdot \underline{U}_l^{(\nu+1)}\right)^* + \underline{U}_i^{(\nu)} \cdot \sum_{l=i}^{n} \left(\underline{Y}_{il} \cdot \underline{U}_l^{(\nu)}\right)^*\right\}$$

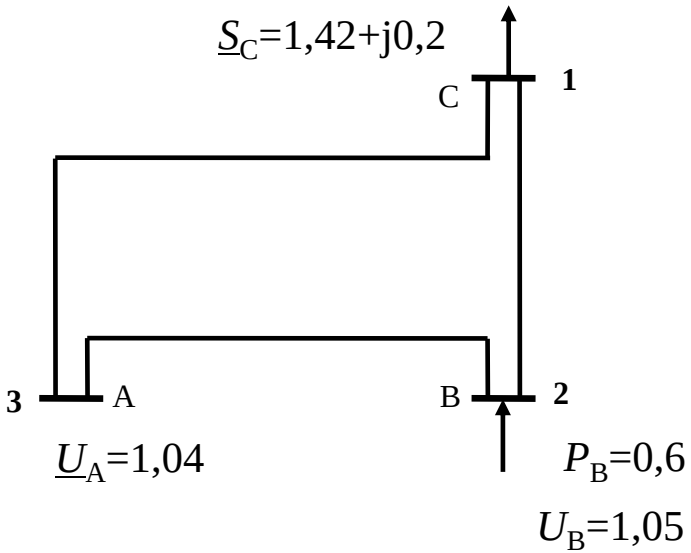
$$Q_i^{(\nu+1)} - Q_{i(\text{пот.})} < Q_{i_{\min}} \Rightarrow Q_i = Q_{i_{\min}} - Q_{i(\text{пот.})} \qquad Q_i^{(\nu+1)} - Q_{i(\text{пот.})} > Q_{i_{\max}} \Rightarrow Q_i = Q_{i_{\max}} - Q_{i(\text{пот.})}$$

$$\underline{U}_i^{(\nu+1)} = \frac{1}{\underline{Y}_{ii}} \cdot \left(\frac{P_i - \mathrm{j}Q_i}{\left(\underline{U}_i^{(\nu)}\right)^*} - \sum_{l=1}^{i-1} \underline{Y}_{il} \cdot \underline{U}_l^{(\nu+1)} - \sum_{l=i+1}^{n} \underline{Y}_{il} \cdot \underline{U}_l^{(\nu)} \right) = E_i^{(\nu+1)} + \mathrm{j}F_i^{(\nu+1)}$$

$$\theta_i^{(\nu+1)} = \arctan\left(\frac{F_i^{(\nu+1)}}{E_i^{(\nu+1)}}\right)$$

$$\underline{U}_i^{(\nu+1)} = U_i \cdot \left(\cos\theta_i^{(\nu+1)} + \mathrm{j}\sin\theta_i^{(\nu+1)}\right)$$

Пример 4.3 Систем со PU



Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,04	0	S	3
C	–1,42	–0,2	?	?	PQ	1
B	0,6	?	1,05	?	PU	2

$$\underline{U}_C^{(0)} = 1{,}0 + \mathrm{j}0{,}0$$

$$\underline{U}_B^{(0)} = 1{,}05 + \mathrm{j}0{,}0$$

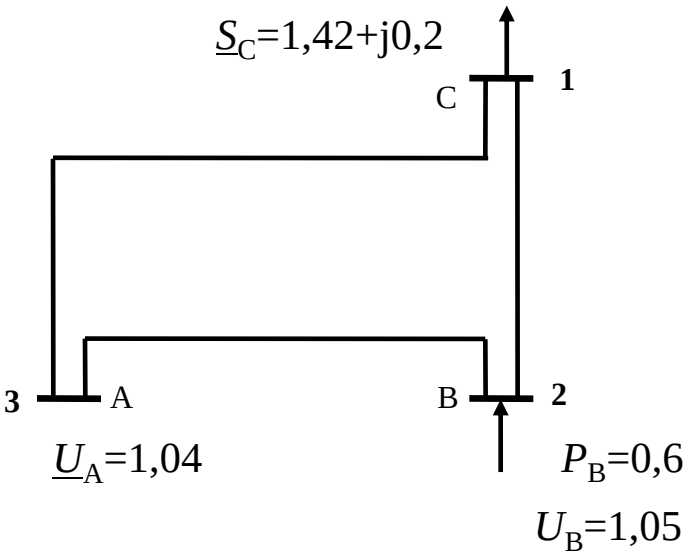
$$\underline{U}_A = 1{,}04$$

Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0,24	0,32	0	0,04
A-C	1-3	0,12	0,16	0	0,02
A-B	2-3	0,24	0,32	0	0,04

$$\underline{\mathbf{Y}} = \begin{array}{c} \\ \mathrm{C} \\ \mathrm{B} \\ \mathrm{A} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{C} & \mathrm{B} & \mathrm{A} \end{array} \\ \begin{bmatrix} 4{,}5 & -1{,}5 & -3{,}0 \\ -1{,}5 & 3{,}0 & -1{,}5 \\ -3{,}0 & -1{,}5 & 4{,}5 \end{bmatrix} \end{array} + \mathrm{j} \begin{array}{c} \begin{array}{ccc} \mathrm{C} & \mathrm{B} & \mathrm{A} \end{array} \\ \begin{bmatrix} -5{,}97 & 2{,}00 & 4{,}00 \\ 2{,}00 & -3{,}96 & 2{,}00 \\ 4{,}00 & 2{,}00 & -5{,}97 \end{bmatrix} \end{array} \begin{array}{c} \mathrm{C} \\ \mathrm{B} \\ \mathrm{A} \end{array}$$

Пример 4.3 Систем со PU



$$\underline{U}_C^{(1)} = \frac{1}{\underline{Y}_{CC}} \cdot \left[\frac{P_C - jQ_C}{\left(\underline{U}_C^{(0)}\right)^*} - \underline{Y}_{CB}\underline{U}_B^{(0)} - \underline{Y}_{CA}\underline{U}_A \right]$$

$$= \frac{\left[\frac{-1,42 - j(-0,2)}{(1+j0)^*} - (-1,5 + j2)\cdot(1,05+j0) - (-3+j4)\cdot(1,04+j0) \right]}{4,5 - j5,97}$$

$$= \frac{3,275 - j6,06}{4,5 - j5,97} = (0,9109837 - j0,1380950)$$

$$\Delta\underline{U}_C^{(1)} = \underline{U}_C^{(1)} - \underline{U}_C^{(0)} = (0,9109837 - j0,1380950) - (1 + j0) = (-0,08901632 - j0,1380950)$$

$$\Delta U_C^{(1)} = 0,1642989 > \varepsilon$$

Пример 4.3 Систем со PU

Пресметка на реактивната моќност во јазолот со познат напон (PU јазол)

• за напонот на јазолот B ја користиме вредноста од претходната итерација

$$Q_i^{(\nu+1)} = \mathrm{Im}\left\{\underline{U}_i \cdot \underline{I}_i^*\right\} = \mathrm{Im}\left\{\underline{U}_i \cdot \sum_{l=1}^{n} (\underline{Y}_{il} \cdot \underline{U}_l)^*\right\} = \mathrm{Im}\left\{\underline{U}_i^{(\nu)} \cdot \sum_{l=1}^{i-1}\left(\underline{Y}_{il} \cdot \underline{U}_l^{(\nu+1)}\right)^* + \underline{U}_i^{(\nu)} \cdot \sum_{l=i}^{n}\left(\underline{Y}_{il} \cdot \underline{U}_l^{(\nu)}\right)^*\right\}$$

$$Q_{\mathrm{B}}^{(1)} = \mathrm{Im}\left\{\underline{U}_{\mathrm{B}}^{(0)} \cdot \underline{Y}_{\mathrm{BC}} \cdot \underline{U}_{\mathrm{C}}^{(1)} + U_{\mathrm{B}}^{(0)^2} \cdot \underline{Y}_{\mathrm{BB}}^* + \underline{U}_{\mathrm{B}}^{(0)} \cdot \underline{Y}_{\mathrm{BA}}^* \cdot \underline{U}_{\mathrm{A}}^*\right\}$$

$$Q_{\mathrm{B}}^{(1)} = \mathrm{Im}\left\{U_{\mathrm{B}}^{(0)^2} \cdot \underline{Y}_{\mathrm{BB}}^* + \underline{U}_{\mathrm{B}}^{(0)} \cdot \left[\underline{Y}_{\mathrm{BC}} \cdot \underline{U}_{\mathrm{C}}^{(1)} + \underline{Y}_{\mathrm{BA}} \cdot \underline{U}_{\mathrm{A}}\right]^*\right\}$$

$$Q_{\mathrm{B}}^{(1)} = \mathrm{Im}\left\{1{,}05^2 \cdot (3 + \mathrm{j}3{,}96) + 1{,}05 \cdot \left[(-1{,}5 + \mathrm{j}2) \cdot (0{,}9109837 - \mathrm{j}0{,}1380950) + (-1{,}5 + \mathrm{j}2) \cdot 1{,}04\right]\right\}$$

$$Q_{\mathrm{B}}^{(1)} = 0{,}05133483$$

Пример 4.3 Систем со PU

$$\underline{U}_{\mathrm{B}}^{(1)} = \frac{1}{\underline{Y}_{\mathrm{BB}}} \cdot \left[\frac{P_{\mathrm{B}} - \mathrm{j}Q_{\mathrm{B}}^{(1)}}{\left(\underline{U}_{\mathrm{B}}^{(0)}\right)^{*}} - \underline{Y}_{\mathrm{BC}}\underline{U}_{\mathrm{C}}^{(1)} - \underline{Y}_{\mathrm{BA}}\underline{U}_{\mathrm{A}} \right]$$

$$= \frac{\left[\frac{0,6 - \mathrm{j}0,05133438}{(1,05+\mathrm{j}0)^{*}} - (-1,5 + \mathrm{j}2) \cdot (0,9109837 - \mathrm{j}0,1380950) - (-1,5+\mathrm{j}2) \cdot (1,04+\mathrm{j}0) \right]}{3 - \mathrm{j}3,96}$$

$$= \frac{0,6 - \mathrm{j}0,05133438 + 1,090286 - \mathrm{j}2,029110 + 1,56 - \mathrm{j}2,08}{3 - \mathrm{j}3,96}$$

$$= \frac{3,221714 - \mathrm{j}4,158}{3 - \mathrm{j}3,96}$$

$$= (1,058717 + \mathrm{j}0,01150605)$$

$$\theta_{\mathrm{B}}^{(1)} = \arctan\left(\frac{F_{\mathrm{B}}^{(1)}}{E_{\mathrm{B}}^{(1)}}\right) = \arctan\left(\frac{0,01150605}{1,058717}\right) = 0,01086749 \text{ rad}$$

$$\underline{U}_{\mathrm{B}}^{(1)} = U_{\mathrm{B}} \cdot \left(\cos\theta_{\mathrm{B}}^{(1)} + \mathrm{j}\sin\theta_{\mathrm{B}}^{(1)}\right)$$

$$= 1,05 \cdot (\cos 0,01086749 + \mathrm{j}\sin 0,01086749)$$

$$= (1,049938 + \mathrm{j}0,01141064)$$

$$\Delta\underline{U}_{\mathrm{B}}^{(1)} = \underline{U}_{\mathrm{B}}^{(1)} - \underline{U}_{\mathrm{B}}^{(0)}$$

$$= (1,049938 + \mathrm{j}0,01141064) - (1,05 + \mathrm{j}0)$$

$$= \left(-6,2 \cdot 10^{-5} + \mathrm{j}0,01141064\right)$$

$$\Delta U_{\mathrm{B}}^{(1)} = 1,1410874 \cdot 10^{-2} > \varepsilon$$

Пример 4.3 Систем со PU

$$\Delta U_{\mathrm{C}}^{(1)} = 0{,}1642989 > \varepsilon \qquad \Delta U_{\mathrm{B}}^{(1)} = 1{,}1410874 \cdot 10^{-2} > \varepsilon$$

$$\underline{U}_{\mathrm{C}}^{(2)} = \frac{1}{\underline{Y}_{\mathrm{CC}}} \cdot \left[\frac{P_{\mathrm{C}} - \mathrm{j}Q_{\mathrm{C}}}{\left(\underline{U}_{\mathrm{C}}^{(1)}\right)^{*}} - \underline{Y}_{\mathrm{CB}}\underline{U}_{\mathrm{B}}^{(1)} - \underline{Y}_{\mathrm{CA}}\underline{U}_{\mathrm{A}} \right]$$

$$Q_{\mathrm{B}}^{(2)} = \mathrm{Im}\left\{ U_{\mathrm{B}}^{(1)^2} \cdot \underline{Y}_{\mathrm{BB}}^{*} + \underline{U}_{\mathrm{B}}^{(1)} \cdot \left[\underline{Y}_{\mathrm{BC}} \cdot \underline{U}_{\mathrm{C}}^{(2)} + \underline{Y}_{\mathrm{BA}} \cdot \underline{U}_{\mathrm{A}} \right]^{*} \right\}$$

$$\underline{U}_{\mathrm{B}}^{(2)} = \frac{1}{\underline{Y}_{\mathrm{BB}}} \cdot \left[\frac{P_{\mathrm{B}} - \mathrm{j}Q_{\mathrm{B}}^{(2)}}{\left(\underline{U}_{\mathrm{B}}^{(1)}\right)^{*}} - \underline{Y}_{\mathrm{BC}}\underline{U}_{\mathrm{C}}^{(2)} - \underline{Y}_{\mathrm{BA}}\underline{U}_{\mathrm{A}} \right]$$

„Точно“ решение после 12 итерации

$$\underline{U}_{\mathrm{C}}^{(12)} = (0{,}8717921 - \mathrm{j}0{,}1294288) = 0{,}88 \cdot \mathrm{e}^{-\mathrm{j}8{,}45^{\circ}}$$

$$\underline{U}_{\mathrm{B}}^{(12)} = (1{,}049970 + \mathrm{j}0{,}007892412) = 1{,}05 \cdot \mathrm{e}^{\mathrm{j}0{,}43^{\circ}}$$

$$Q_{\mathrm{B}} = 0{,}1268152$$

Пример 4.3 Систем со PU

„точно“ решение после 12 итерации:

```
 Vlezni podatoci za grankite
-------------------------------------------------------------------------------
           R             X        G'xE-06       B'xE-06       Prenosen odnos
-------------------------------------------------------------------------------
 A - B 0.2400000     0.3200000      0.000000      40000.00        1.00/   1.00
 B - C 0.2400000     0.3200000      0.000000      40000.00        1.00/   1.00
 A - C 0.1200000     0.1600000      0.000000      20000.00        1.00/   1.00
-------------------------------------------------------------------------------

     Vlezni podatoci za jazlite
-------------------------------------------------------------------------
Jazol     P              Q           U             Theta       Tip   Ubaz
-------------------------------------------------------------------------
   C -1.420000     -0.2000000        ?              ?           PQ   1.00
   B 0.6000000          ?          1.050000        ?           PV   1.00
   A    ?               ?          1.040000      0.000000       S   1.00
-------------------------------------------------------------------------
 Parametri od pi-ekv. semi
 3-2    Y,Y'/2(  1.500000     j -2.000000    )(  0.000000     j 0.2000000E-01)
 2-1    Y,Y'/2(  1.500000     j -2.000000    )(  0.000000     j 0.2000000E-01)
 3-1    Y,Y'/2(  3.000000     j -4.000000    )(  0.000000     j 0.1000000E-01)
Formiranje na matricata Y so inspekcija na mrezata
      -------------------------------------------------
           Matrica G                                  Matrica B
        C             B             A              C             B             A
 C  4.500000     -1.500000     -3.000000     C -5.970000      2.000000      4.000000
 B -1.500000      3.000000     -1.500000     B  2.000000     -3.960000      2.000000
 A -3.000000     -1.500000      4.500000     A  4.000000      2.000000     -5.970000
```

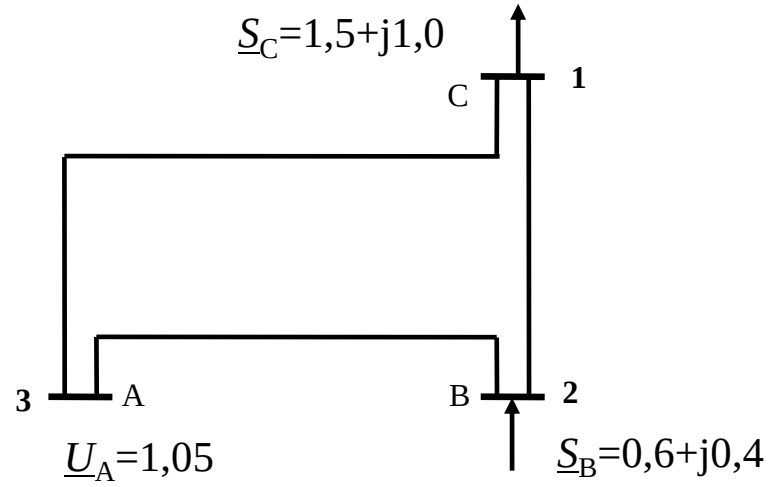
Пример 4.3 Систем со PU

```
Resenie so Gauss-Seidel-ov metod; alfa=   1.000000     ; eps=  1.0000001E-05

Naponi vo jazlite posle  12.0 iteracii; (dU)max=    0.5779788E-05 < 0.1000000E-04
-------------------------------------------------------------------------------------------
   (      E       +j      F       )  (    U          [ejth (rad.)]    )  (        U[ejth]      )
-------------------------------------------------------------------------------------------
 C ( 0.8717921     j-0.1294288    )  ( 0.8813475     ej-0.1473864    )  (     0.88 ej -8.44462)
 B (  1.049970     j 0.7892406E-02)  (  1.050000     ej 0.7516648E-02)  (     1.05 ej  0.43067)
 A (  1.040000     j  0.000000    )  (  1.040000     ej  0.000000    )  (     1.04 ej  0.00000)
-------------------------------------------------------------------------------------------

                    Zadadeni i presmetani moknosti vo jazlite (p.u.)
-------------------------------------------------------------------------------------------
        (      P       +j      Q       )(      P       +j      Q       )(     dP       +j      dQ      )
-------------------------------------------------------------------------------------------
C(PQ)   ( -1.420000     j-0.2000000    )( -1.419991     j-0.2000037    )(-0.9417534E-05 j 0.3725290E-05)
B(PV)   ( 0.6000000     j  0.000000    )( 0.6000146     j 0.1268152    )(-0.1460314E-04 j  0.000000    )
A(S )   (  0.000000     j  0.000000    )(  1.031262     j 0.2550521    )(  0.000000     j  0.000000    )
-------------------------------------------------------------------------------------------
Zagubi: ( 0.2112861     j 0.1818636    )
```

Пример 4.4 Систем без PU јазли



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0	0,32	0	0,04
A-C	1-3	0	0,16	0	0,02
A-B	2-3	0	0,08	0	0,01

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,05	0	S	3
C	–1,5	–1,0	?	?	PQ	1
B	0,6	0,4	?	?	PQ	2

$$\underline{Y}_{A-B} = \frac{1}{\underline{Z}_{A-B}} = \frac{1}{j0,32} = -j12,5$$

$$\underline{Y}_{C-A} = \frac{1}{\underline{Z}_{C-A}} = \frac{1}{j0,16} = -j6,25$$

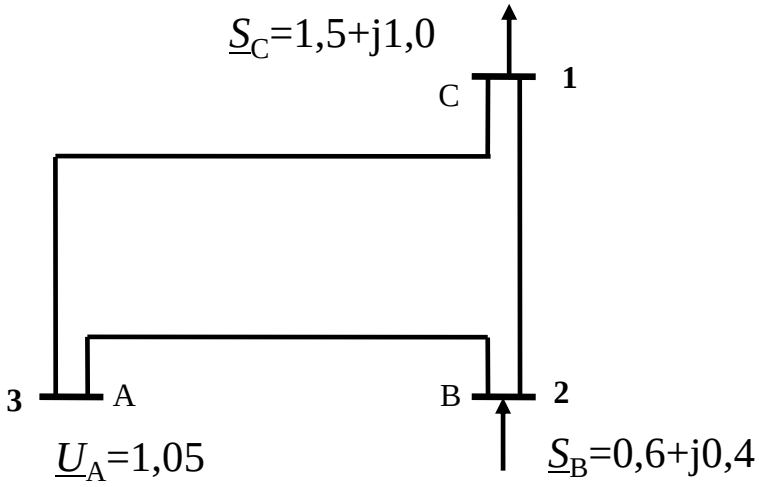
$$\underline{Y}_{B-C} = \frac{1}{\underline{Z}_{B-C}} = \frac{1}{j0,32} = -j3,125$$

$$U_C^{(0)} = 1,0 \quad U_B^{(0)} = 1,0 \quad U_A = 1,05$$

$$\theta_C^{(0)} = 0 \qquad \theta_B^{(0)} = 0 \qquad \theta_A = 0$$

$$n-1=2;\ q=2$$

Пример 4.4 Систем без PU јазли



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0	0,32	0	0,04
A-C	1-3	0	0,16	0	0,02
A-B	2-3	0	0,08	0	0,01

$$\underline{Y}_{A-B} = \frac{1}{\underline{Z}_{A-B}} = \frac{1}{j0,32} = -j12,5$$

$$\underline{Y}_{C-A} = \frac{1}{\underline{Z}_{C-A}} = \frac{1}{j0,16} = -j6,25$$

$$\underline{Y}_{B-C} = \frac{1}{\underline{Z}_{B-C}} = \frac{1}{j0,32} = -j3,125$$

$$\underline{Y}_{AA} = \underline{Y}_{C-A} + \underline{Y}_{A-B} + \tfrac{1}{2}\cdot\left(\underline{Y}'_{C-A} + \underline{Y}'_{A-B}\right)$$

$$= -j6,25 - j12,5 + \tfrac{1}{2}\cdot(j0,02 + j0,01) = j18,735$$

$$\underline{Y}_{BB} = \underline{Y}_{B-C} + \underline{Y}_{A-B} + \tfrac{1}{2}\cdot\left(\underline{Y}'_{B-C} + \underline{Y}'_{A-B}\right)$$

$$= -j3,125 - j12,5 + \tfrac{1}{2}\cdot(j0,04 + j0,01) = -j15,6$$

$$\underline{Y}_{CC} = \underline{Y}_{C-A} + \underline{Y}_{B-C} + \tfrac{1}{2}\cdot\left(\underline{Y}'_{C-A} + \underline{Y}'_{B-C}\right)$$

$$= -j6,25 - j3,125 + \tfrac{1}{2}\cdot(j0,02 + j0,04) = -j9,345$$

$$\underline{Y}_{CA} = \underline{Y}_{AC} = -\underline{Y}_{C-A} = -(-j6,25) = j6,25$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{B-C} = -(-j3,125) = j3,125$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = -\underline{Y}_{A-B} = -(-j12,5) = j12,5$$

$$\underline{\boldsymbol{Y}} = j\begin{array}{c} \\ C \\ B \\ A \end{array}\begin{array}{c} \begin{array}{ccc} C & B & A \end{array} \\ \begin{bmatrix} -9,345 & 3,125 & 6,250 \\ 3,125 & -15,600 & 12,500 \\ 6,250 & 12,500 & -18,735 \end{bmatrix} \end{array}$$

Пример 4.4 Систем без PU јазли

Пресметани инјектирани моќности во јазлите пред првата итерација:

$$\theta_{\mathrm{CB}} = \theta_{\mathrm{C}}^{(0)} - \theta_{\mathrm{B}}^{(0)} = 0 \qquad \theta_{\mathrm{CA}} = \theta_{\mathrm{C}}^{(0)} - \theta_{\mathrm{A}} = 0$$

$$P_{\mathrm{C}_{\text{пресметана}}}^{(0)} = G_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \cos\theta_{ik} + B_{ik} \cdot \sin\theta_{ik} \right)$$

$$= G_{\mathrm{CC}} \cdot U_{\mathrm{C}}^{(0)^2} + U_{\mathrm{C}}^{(0)} \cdot \left[U_{\mathrm{B}}^{(0)} \cdot \left(G_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}} + B_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{CA}} \cdot \cos\theta_{\mathrm{CA}} + B_{\mathrm{CA}} \cdot \sin\theta_{\mathrm{CA}} \right) \right]$$

$$= 0 \cdot 1{,}0^2 + 1{,}0 \cdot \left[1{,}0 \cdot \left(0 \cdot 1 + 3{,}125 \cdot 0 \right) + 1{,}05 \cdot \left(0 \cdot 1 + 6{,}25 \cdot 0 \right) \right] = 0 \qquad \leftarrow \underline{\boldsymbol{Y}} = \mathrm{j}\boldsymbol{B}$$

$$\theta_{\mathrm{BC}} = \theta_{\mathrm{B}}^{(0)} - \theta_{\mathrm{C}}^{(0)} = 0 \quad \theta_{\mathrm{BA}} = \theta_{\mathrm{B}}^{(0)} - \theta_{\mathrm{A}} = 0$$

$$P_{\mathrm{B}_{\text{пресметана}}}^{(0)} = G_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \cos\theta_{ik} + B_{ik} \cdot \sin\theta_{ik} \right)$$

$$= G_{\mathrm{BB}} \cdot U_{\mathrm{B}}^{(0)^2} + U_{\mathrm{B}}^{(0)} \cdot \left[U_{\mathrm{C}}^{(0)} \cdot \left(G_{\mathrm{BC}} \cdot \cos\theta_{\mathrm{BC}} + B_{\mathrm{BC}} \cdot \sin\theta_{\mathrm{BC}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{BA}} \cdot \cos\theta_{\mathrm{BA}} + B_{\mathrm{BA}} \cdot \sin\theta_{\mathrm{BA}} \right) \right]$$

$$= 0 \cdot 1{,}0^2 + 1{,}0 \cdot \left[1{,}0 \cdot \left(0 \cdot 1 + 3{,}125 \cdot 0 \right) + 1{,}05 \cdot \left(0 \cdot 0 + 12{,}5 \cdot 0 \right) \right] = 0 \qquad \leftarrow \underline{\boldsymbol{Y}} = \mathrm{j}\boldsymbol{B}$$

Пример 4.4 Систем без PU јазли

Пресметани инјектирани моќности во јазлите пред првата итерација:

$$\theta_{\mathrm{CB}} = \theta_{\mathrm{C}}^{(0)} - \theta_{\mathrm{B}}^{(0)} = 0$$
$$\theta_{\mathrm{CA}} = \theta_{\mathrm{C}}^{(0)} - \theta_{\mathrm{A}} = 0$$

$$Q_{\mathrm{C}_{\text{пресметана}}}^{(0)} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik} \right)$$

$$= -B_{\mathrm{CC}} \cdot U_{\mathrm{C}}^{(0)^2} + U_{\mathrm{C}}^{(0)} \cdot \left[U_{\mathrm{B}}^{(0)} \cdot \left(G_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} - B_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{CA}} \cdot \sin\theta_{\mathrm{CA}} - B_{\mathrm{CA}} \cdot \cos\theta_{\mathrm{CA}} \right) \right]$$

$$= -\left(-9,345\right) \cdot 1,0^2 + 1,0 \cdot \left[1,0 \cdot \left(0 \cdot 0 - 3,125 \cdot 1 \right) + 1,05 \cdot \left(0 \cdot 0 - 6,25 \cdot 1 \right) \right] = -0,3425$$

$$\theta_{\mathrm{BC}} = \theta_{\mathrm{B}}^{(0)} - \theta_{\mathrm{C}}^{(0)} = 0$$
$$\theta_{\mathrm{BA}} = \theta_{\mathrm{B}}^{(0)} - \theta_{\mathrm{A}} = 0$$

$$Q_{\mathrm{B}_{\text{пресметана}}}^{(0)} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik} \right)$$

$$= -B_{\mathrm{BB}} \cdot U_{\mathrm{B}}^{(0)^2} + U_{\mathrm{B}}^{(0)} \cdot \left[U_{\mathrm{C}}^{(0)} \cdot \left(G_{\mathrm{BC}} \cdot \sin\theta_{\mathrm{BC}} - B_{\mathrm{BC}} \cdot \cos\theta_{\mathrm{BC}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{BA}} \cdot \sin\theta_{\mathrm{BA}} - B_{\mathrm{BA}} \cdot \cos\theta_{\mathrm{BA}} \right) \right]$$

$$= -\left(-15,6\right) \cdot 1,0^2 + 1,0 \cdot \left[1,0 \cdot \left(0 \cdot 0 - 3,125 \cdot 1 \right) + 1,05 \cdot \left(0 \cdot 0 - 12,5 \cdot 1 \right) \right] = -0,65$$

Пример 4.4 Систем без PU јазли

Пресметани инјектирани моќности во јазлите пред првата итерација:

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,05	0	S	3
C	–1,5	–1,0	?	?	PQ	1
B	0,6	0,4	?	?	PQ	2

$$\Delta P_{\mathrm{C}}^{(0)} = P_{\mathrm{C}_{\text{зададена}}} - P_{\mathrm{C}_{\text{пресметана}}}^{(0)} = -1,5 - 0 = -1,5$$

$$\Delta P_{\mathrm{B}}^{(0)} = P_{\mathrm{B}_{\text{зададена}}} - P_{\mathrm{B}_{\text{пресметана}}}^{(0)} = 0,6 - 0 = 0,6$$

$$\Delta Q_{\mathrm{C}}^{(0)} = Q_{\mathrm{C}_{\text{зададена}}} - Q_{\mathrm{C}_{\text{пресметана}}}^{(0)} = -1,0 - \left(-0,3425\right) = -0,6575$$

$$\Delta Q_{\mathrm{B}}^{(0)} = Q_{\mathrm{B}_{\text{зададена}}} - Q_{\mathrm{B}_{\text{пресметана}}}^{(0)} = 0,4 - \left(-0,65\right) = 1,05$$

Пример 4.4 Систем без PU јазли

Пресметка на елементите од јакобијанот

$$\begin{bmatrix} \boldsymbol{H} & \boldsymbol{N} \\ \boldsymbol{M} & \boldsymbol{L} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta} \\ \Delta\boldsymbol{U}/\boldsymbol{U} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P} \\ \Delta\boldsymbol{Q} \end{bmatrix}$$

$$n = 3 \rightarrow (n-1 = 2)$$

$$q = 2$$

$$\begin{bmatrix} \boldsymbol{H}_{2\times2} & \boldsymbol{N}_{2\times2} \\ \boldsymbol{M}_{2\times2} & \boldsymbol{L}_{2\times2} \end{bmatrix}_{4\times4} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta}_{2\times1} \\ (\Delta\boldsymbol{U}/\boldsymbol{U})_{2\times1} \end{bmatrix}_{4\times1} = \begin{bmatrix} \Delta\boldsymbol{P}_{2\times1} \\ \Delta\boldsymbol{Q}_{2\times1} \end{bmatrix}_{4\times1}$$

$$\begin{bmatrix} H_{CC} & H_{CB} & N_{CC} & N_{CB} \\ H_{BC} & H_{BB} & N_{BC} & N_{BB} \\ M_{CC} & M_{CB} & L_{CC} & L_{CB} \\ M_{BC} & M_{BB} & L_{BC} & L_{BB} \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_C \\ \Delta\theta_B \\ \dfrac{\Delta U_C}{U_C} \\ \dfrac{\Delta U_B}{U_B} \end{bmatrix} = \begin{bmatrix} \Delta P_C \\ \Delta P_B \\ \Delta Q_C \\ \Delta Q_B \end{bmatrix}$$

Пример 4.4 Систем без PU јазли

Пресметка на елементите од јакобијанот

$$\begin{bmatrix} H_{CC} & H_{CB} & N_{CC} & N_{CB} \\ H_{BC} & H_{BB} & N_{BC} & N_{BB} \\ M_{CC} & M_{CB} & L_{CC} & L_{CB} \\ M_{BC} & M_{BB} & L_{BC} & L_{BB} \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_C \\ \Delta\theta_B \\ \dfrac{\Delta U_C}{U_C} \\ \dfrac{\Delta U_B}{U_B} \end{bmatrix} = \begin{bmatrix} \Delta P_C \\ \Delta P_B \\ \Delta Q_C \\ \Delta Q_B \end{bmatrix}$$

$$H_{ij} = \frac{\partial P_i}{\partial \theta_j} = U_i \cdot U_j \cdot \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right)$$

$$H_{CB} = L_{CB} = U_C \cdot U_B \cdot (G_{CB} \cdot \sin\theta_{CB} - B_{CB} \cdot \cos\theta_{CB}) = 1{,}0 \cdot 1{,}0 \cdot (0 - 3{,}125) = -3{,}125$$

$$H_{BC} = L_{BC} = U_B \cdot U_C \cdot (G_{BC} \cdot \sin\theta_{BC} - B_{BC} \cdot \cos\theta_{BC}) = 1{,}0 \cdot 1{,}0 \cdot (0 - 3{,}125) = -3{,}125$$

Субматрицата *H* е симетрична само во нултата итерација затоа што $\theta_{ij} = \theta_{ji} = 0$!

$$L_{ij} = H_{ij} \quad ; \quad j \neq i$$

$$L_{CB} = -3{,}125 \qquad L_{BC} = -3{,}125$$

Пример 4.4 Систем без PU јазли

Пресметка на елементите од јакобијанот

$$\begin{bmatrix} H_{CC} & H_{CB} & N_{CC} & N_{CB} \\ H_{BC} & H_{BB} & N_{BC} & N_{BB} \\ M_{CC} & M_{CB} & L_{CC} & L_{CB} \\ M_{BC} & M_{BB} & L_{BC} & L_{BB} \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_C \\ \Delta\theta_B \\ \frac{\Delta U_C}{U_C} \\ \frac{\Delta U_B}{U_B} \end{bmatrix} = \begin{bmatrix} \Delta P_C \\ \Delta P_B \\ \Delta Q_C \\ \Delta Q_B \end{bmatrix}$$

$$H_{ii} = \frac{\partial P_i}{\partial \theta_i} = -B_{ii} \cdot U_i^2 - Q_{i_{\text{пресметана}}}$$

$$H_{CC} = -B_{CC} \cdot U_C^2 - Q_{C_{\text{пресметана}}} = -\left(-9,345\right) \cdot 1^2 - \left(-0,3425\right) = 9,6875$$

$$H_{BB} = -B_{BB} \cdot U_{BB}^2 - Q_{B_{\text{пресметана}}} = -\left(-15,6\right) \cdot 1^2 - \left(-0,65\right) = 16,25$$

$$L_{ii} = \frac{\partial Q_i}{\partial U_i} U_i = -B_{ii} \cdot U_i^2 + Q_{i_{\text{пресметана}}}$$

$$L_{CC} = -B_{CC} \cdot U_C^2 + Q_{C_{\text{пресметана}}} = -\left(-9,345\right) \cdot 1^2 + \left(-0,3425\right) = 9,025$$

$$L_{BB} = -B_{BB} \cdot U_{BB}^2 + Q_{B_{\text{пресметана}}} = -\left(-15,6\right) \cdot 1^2 + \left(-0,65\right) = 14,95$$

Пример 4.4 Систем без PU јазли

$$\begin{bmatrix} H_{CC} & H_{CB} & N_{CC} & N_{CB} \\ H_{BC} & H_{BB} & N_{BC} & N_{BB} \\ M_{CC} & M_{CB} & L_{CC} & L_{CB} \\ M_{BC} & M_{BB} & L_{BC} & L_{BB} \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_C \\ \Delta\theta_B \\ \dfrac{\Delta U_C}{U_C} \\ \dfrac{\Delta U_B}{U_B} \end{bmatrix} = \begin{bmatrix} \Delta P_C \\ \Delta P_B \\ \Delta Q_C \\ \Delta Q_B \end{bmatrix}$$

$$\underline{\boldsymbol{Y}} = \boldsymbol{G} + \mathrm{j}\boldsymbol{B}$$

$$N_{ii} = \frac{\partial P_i}{\partial U_i} U_i = G_{ii} \cdot U_i^2 + P_{i_{\text{пресметана}}} = 0$$

$$N_{ij} = \frac{\partial P_i}{\partial U_j} U_j = U_i \cdot U_j \cdot \left(G_{ij} \cdot \cos\theta_{ij} + B_{ij} \cdot \sin\theta_{ij} \right) = 0$$

$$M_{ii} = \frac{\partial Q_i}{\partial \theta_i} = -G_{ii} \cdot U_i^2 + P_{i_{\text{пресметана}}} = 0$$

$$M_{ij} = -N_{ij} = 0$$

$$\left[\begin{array}{cc:cc} 9,6875 & -3,125 & 0 & 0 \\ -3,125 & 16,25 & 0 & 0 \\ \hdashline 0 & 0 & 9,025 & -3,125 \\ 0 & 0 & -3,125 & 14,95 \end{array}\right] \cdot \left[\begin{array}{c} \Delta\theta_C^{(0)} \\ \Delta\theta_B^{(0)} \\ \hdashline \left(\Delta U_C / U_C\right)^{(0)} \\ \left(\Delta U_B / U_B\right)^{(0)} \end{array}\right] = \left[\begin{array}{c} -1,5 \\ 0,6 \\ \hdashline -0,6575 \\ 1,05 \end{array}\right]$$

Пример 4.4 Систем без PU јазли

Гаусова елиминација

Бидејќи субматриците ***N*** и ***M*** се еднакви на нула, решаваме два независни система линеарни равенки со по две непознати

1.a $\begin{bmatrix} 1 & -0{,}3225806 \\ -3{,}125 & 16{,}25 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}1548387 \\ 0{,}6 \end{bmatrix}$ $\begin{bmatrix} 1 & -0{,}3471259 \\ -3{,}125 & 14{,}95 \end{bmatrix} \cdot \begin{bmatrix} (\Delta U_{\mathrm{C}}/U_{\mathrm{C}})^{(0)} \\ (\Delta U_{\mathrm{B}}/U_{\mathrm{B}})^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}0730352 \\ 1{,}05 \end{bmatrix}$

1.b $\begin{bmatrix} 1 & -0{,}3225806 \\ 0 & 15{,}24194 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}1548387 \\ 0{,}1161291 \end{bmatrix}$ $\begin{bmatrix} 1 & -0{,}3471259 \\ 0 & 13{,}86523 \end{bmatrix} \cdot \begin{bmatrix} \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \\ \left(\Delta U_{\mathrm{B}}/U_{\mathrm{B}}\right)^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}0730352 \\ 0{,}8217655 \end{bmatrix}$

2.a $\begin{bmatrix} 1 & -0{,}3225806 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}1548387 \\ 0{,}0076190 \end{bmatrix}$ $\begin{bmatrix} 1 & -0{,}3471259 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \\ \left(\Delta U_{\mathrm{B}}/U_{\mathrm{B}}\right)^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}0730352 \\ 0{,}0592681 \end{bmatrix}$

Пример 4.4 Систем без PU јазли

$$\begin{bmatrix} 1 & -0{,}3225806 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}1548387 \\ 0{,}0076190 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0{,}3471259 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \\ \left(\Delta U_{\mathrm{B}}/U_{\mathrm{B}}\right)^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}0730352 \\ 0{,}0592681 \end{bmatrix}$$

Повратна замена

$$\begin{aligned} \Delta\theta_{\mathrm{C}}^{(0)} &= -0{,}1548387 - \Delta\theta_{\mathrm{B}}^{(0)} \cdot (-0{,}3225806) \\ &= -0{,}1548387 - 0{,}0076190 \cdot (-0{,}3225806) \\ &= -0{,}1523810 \end{aligned}$$

$$\begin{aligned} \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} &= -0{,}0730352 - \left(\Delta U_{\mathrm{B}}/U_{\mathrm{B}}\right)^{(0)} \cdot 0{,}0592681 \\ &= -0{,}0730352 - (-0{,}0730352) \cdot 0{,}0592681 \\ &= -0{,}0524617 \end{aligned}$$

Напони на јазлите после првата итерација:

$$\theta_{\mathrm{C}}^{(1)} = \theta_{\mathrm{C}}^{(0)} + \Delta\theta_{\mathrm{C}}^{(0)} = 0 + \Delta\theta_{\mathrm{C}}^{(0)} = -0{,}1523810 \text{ rad}$$

$$\theta_{\mathrm{B}}^{(1)} = \theta_{\mathrm{B}}^{(0)} + \Delta\theta_{\mathrm{B}}^{(0)} = 0 + \Delta\theta_{\mathrm{B}}^{(0)} = 0{,}0076190 \text{ rad}$$

$$\begin{aligned} U_{\mathrm{C}}^{(1)} &= U_{\mathrm{C}}^{(0)} + \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \cdot U_{\mathrm{C}}^{(0)} = 1{,}0 + (-0{,}0524617) \cdot 1{,}0 \\ &= 0{,}9475383 \end{aligned}$$

$$\begin{aligned} U_{\mathrm{B}}^{(1)} &= U_{\mathrm{B}}^{(0)} + (\Delta U_{\mathrm{B}}/U_{\mathrm{B}})^{(0)} \cdot U_{\mathrm{B}}^{(0)} = 1{,}0 + 0{,}0592681 \cdot 1{,}0 \\ &= 1{,}059268 \end{aligned}$$

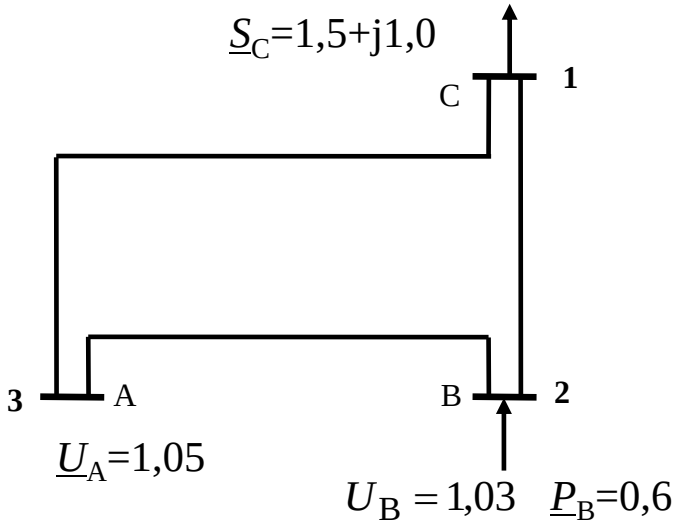
Точно решение (e=0,001) после три итерации

$$\max\left(\left|\Delta P^{(3)}\right|, \left|\Delta Q^{(3)}\right|\right) = 6{,}735325 \cdot 10^{-06}$$

$$\underline{U}_{\mathrm{C}}^{(3)} = 0{,}9221041 \cdot \mathrm{e}^{-\mathrm{j}9{,}4^{\circ}}$$

$$\underline{U}_{\mathrm{B}}^{(3)} = 1{,}04784 \cdot \mathrm{e}^{\mathrm{j}0{,}37^{\circ}}$$

Пример 4.5 Систем со PU јазли



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0	0,32	0	0,04
A-C	1-3	0	0,16	0	0,02
A-B	2-3	0	0,08	0	0,01

$$\underline{\boldsymbol{Y}} = \mathrm{j}\begin{bmatrix} -9{,}345 & 3{,}125 & 6{,}250 \\ 3{,}125 & -15{,}600 & 12{,}500 \\ 6{,}250 & 12{,}500 & -18{,}735 \end{bmatrix} \begin{matrix} \mathrm{C} \\ \mathrm{B} \\ \mathrm{A} \end{matrix}$$

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,05	0	S	3
C	–1,5	–1,0	?	?	PQ	1
B	0,6	?	1,03	?	PU	2

$$\theta_C^{(0)} = 0 \qquad U_A = 1{,}05$$

$$\theta_B^{(0)} = 0 \qquad \theta_A = 0$$

$$n-1=2;\ \mathrm{q}=1 \qquad U_B = 1{,}03$$

$$U_C^{(0)} = 1{,}0$$

$$\begin{bmatrix} \boldsymbol{H}_{2\times 2} & \boldsymbol{N}_{2\times 1} \\ \boldsymbol{M}_{1\times 2} & \boldsymbol{L}_{1\times 1} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta}_{2\times 1} \\ (\Delta \boldsymbol{U}/\boldsymbol{U})_{1\times 1} \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{P}_{2\times 1} \\ \Delta \boldsymbol{Q}_{1\times 1} \end{bmatrix}$$

Пример 4.5 Систем со PU јазли

Пресметани инјектирани моќности во јазлите пред првата итерација:

$$P^{(0)}_{\mathrm{C}_{\text{пресметана}}} = 0 \qquad \Delta P^{(0)}_{\mathrm{C}} = P_{\mathrm{C}_{\text{дадена}}} - P^{(0)}_{\mathrm{C}_{\text{пресметана}}} = -1,5 - 0 = -1,5$$

$$P^{(0)}_{\mathrm{B}_{\text{пресметана}}} = 0 \qquad \mathrm{D}P^{(0)}_{\mathrm{B}} = P_{\mathrm{B}_{\text{дадена}}} - P^{(0)}_{\mathrm{B}_{\text{пресметана}}} = 0,6 - 0 = 0,6$$

$$\underline{\boldsymbol{Y}} = \mathrm{j}\begin{array}{c} \\ \mathrm{C} \\ \mathrm{B} \\ \mathrm{A} \end{array}\begin{array}{c} \begin{array}{ccc} \mathrm{C} & \mathrm{B} & \mathrm{A} \end{array} \\ \begin{bmatrix} -9,345 & 3,125 & 6,250 \\ 3,125 & -15,600 & 12,500 \\ 6,250 & 12,500 & -18,735 \end{bmatrix} \end{array}$$

$$\theta_{\mathrm{CB}} = \theta^{(0)}_{\mathrm{C}} - \theta^{(0)}_{\mathrm{B}} = 0$$

$$\theta_{\mathrm{CA}} = \theta^{(0)}_{\mathrm{C}} - \theta_{\mathrm{A}} = 0$$

$$Q^{(0)}_{\mathrm{C}_{\text{пресметана}}} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik} \right)$$

$$= -B_{\mathrm{CC}} \cdot U_{\mathrm{C}}^{(0)^2} + U_{\mathrm{C}}^{(0)} \cdot \left[U_{\mathrm{B}} \cdot \left(G_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} - B_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{CA}} \cdot \sin\theta_{\mathrm{CA}} - B_{\mathrm{CA}} \cdot \cos\theta_{\mathrm{CA}} \right) \right]$$

$$= -\left(-9,345\right) \cdot 1,0^2 + 1,0 \cdot \left[1,03 \cdot \left(0 \cdot 0 - 3,125 \cdot 1 \right) + 1,05 \cdot \left(0 \cdot 0 - 6,25 \cdot 1 \right) \right] = -0,4362507$$

$$\Delta Q^{(0)}_{\mathrm{C}} = Q_{\mathrm{C}_{\text{дадена}}} - Q^{(0)}_{\mathrm{C}_{\text{пресметана}}} = -1,0 - \left(-0,4362507\right) = -0,5637493$$

Пример 4.5 Систем со PU јазли

Пресметка на елементите од јакобијанот

$$H_{ij} = L_{ij} \quad ; \quad j \neq i$$

$$H_{ij} = \frac{\partial P_i}{\partial \theta_j} = U_i \cdot U_j \cdot \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right)$$

$$\theta_{\mathrm{BC}} = \theta_{\mathrm{B}}^{(0)} - \theta_{\mathrm{C}}^{(0)} = 0$$

$$\theta_{\mathrm{BA}} = \theta_{\mathrm{B}}^{(0)} - \theta_{\mathrm{A}} = 0$$

$$H_{\mathrm{CB}} = U_{\mathrm{C}} \cdot U_{\mathrm{B}} \cdot (G_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} - B_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}}) = 1{,}0 \cdot 1{,}03 \cdot (0 - 3{,}125) = -3{,}21875$$

$$H_{\mathrm{BC}} = U_{\mathrm{B}} \cdot U_{\mathrm{C}} \cdot (G_{\mathrm{BC}} \cdot \sin\theta_{\mathrm{BC}} - B_{\mathrm{BC}} \cdot \cos\theta_{\mathrm{BC}}) = 1{,}03 \cdot 1{,}0 \cdot (0 - 3{,}125) = -3{,}21875$$

Пример 4.5 Систем со PU јазли

Пресметка на елементите од јакобијанот

$$H_{\text{ВВ}} = -B_{\text{ВВ}} \cdot U_{\text{ВВ}}^2 - Q_{\text{В}_{\text{пресметана}}}$$

$$\theta_{\text{ВС}} = \theta_{\text{В}}^{(0)} - \theta_{\text{С}}^{(0)} = 0$$

$$\theta_{\text{ВА}} = \theta_{\text{В}}^{(0)} - \theta_{\text{А}} = 0$$

$$Q_{\text{В}_{\text{пресметана}}}^{(0)} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik}\right)$$

$$= -B_{\text{ВВ}} \cdot U_{\text{В}}^2 + U_{\text{В}} \cdot \left[U_{\text{С}}^{(0)} \cdot \left(G_{\text{ВС}} \cdot \sin\theta_{\text{ВС}} - B_{\text{ВС}} \cdot \cos\theta_{\text{ВС}}\right) + U_{\text{А}} \cdot \left(G_{\text{ВА}} \cdot \sin\theta_{\text{ВА}} - B_{\text{ВА}} \cdot \cos\theta_{\text{ВА}}\right)\right]$$

$$= -\left(-15,6\right) \cdot 1,03^2 + 1,03 \cdot \left[1,0 \cdot \left(0 \cdot 0 - 3,125 \cdot 1\right) + 1,05 \cdot \left(0 \cdot 0 - 12,5 \cdot 1\right)\right] = -0,1874602$$

$$H_{\text{ВВ}} = -B_{\text{ВВ}} \cdot U_{\text{ВВ}}^2 - Q_{\text{В}_{\text{пресметана}}} = -\left(-15,6\right) \cdot 1,03^2 - \left(-0,1874602\right) = 16,73750$$

$$L_{ii} = \frac{\partial Q_i}{\partial U_i} U_i = -B_{ii} \cdot U_i^2 + Q_{i_{\text{пресметана}}}$$

$$L_{\text{СС}} = -B_{\text{СС}} \cdot U_{\text{С}}^2 + Q_{\text{С}_{\text{пресметана}}} = -\left(-9,345\right) \cdot 1^2 + \left(-0,4362507\right) = 8,908749$$

Пример 4.5 Систем со PU јазли

Пресметка на елементите од јакобијанот

$\underline{\boldsymbol{Y}} = \mathrm{j}\boldsymbol{B}$

$$N_{ii} = \frac{\partial P_i}{\partial U_i} U_i = G_{ii} \cdot U_i^2 + P_{i_{\text{пресметана}}} = 0 \qquad N_{ij} = \frac{\partial P_i}{\partial U_j} U_j = U_i \cdot U_j \cdot \left(G_{ij} \cdot \cos\theta_{ij} + B_{ij} \cdot \sin\theta_{ij} \right) = 0$$

$$M_{ii} = \frac{\partial Q_i}{\partial \theta_i} = -G_{ii} \cdot U_i^2 + P_{i_{\text{пресметана}}} = 0 \qquad M_{ij} = -N_{ij} = 0$$

$$\left[\begin{array}{cc|c} 9{,}78125 & -3{,}21875 & 0 \\ -3{,}21875 & 16{,}7375 & 0 \\ \hline 0 & 0 & 8{,}908749 \end{array}\right] \cdot \left[\begin{array}{c} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \\ \hline \left(\Delta U_{\mathrm{C}} / U_{\mathrm{C}}\right)^{(0)} \end{array}\right] = \left[\begin{array}{c} -1{,}5 \\ 0{,}6 \\ \hline -0{,}5637493 \end{array}\right]$$

Пример 4.5 Систем со PU јазли

Пресметка на елементите од јакобијанот

$$\left[\begin{array}{cc:c} 9{,}78125 & -3{,}21875 & 0 \\ -3{,}21875 & 16{,}7375 & 0 \\ \hdashline 0 & 0 & 8{,}908749 \end{array}\right] \cdot \left[\begin{array}{c} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \\ \hdashline \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \end{array}\right] = \left[\begin{array}{c} -1{,}5 \\ 0{,}6 \\ \hdashline -0{,}5637493 \end{array}\right]$$

$$\left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} = \frac{-0{,}5637493}{8{,}908749} = -0{,}0632804$$

$$U_{\mathrm{C}}^{(1)} = U_{\mathrm{C}}^{(0)} \cdot \left[1 + \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)}\right] = 1{,}0 \cdot \left[1 + \left(-0{,}0632804\right)\right] = 0{,}9367196$$

Пример 4.5 Систем со PU јазли

Решавање на системот равенки

$$\begin{bmatrix} 1 & -0{,}3290735 \\ 0 & 15{,}67829 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}1533546 \\ 0{,}1063898 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0{,}3290735 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}1533546 \\ 0{,}0067858 \end{bmatrix}$$

$$\begin{aligned} \Delta\theta_{\mathrm{C}}^{(0)} &= -0{,}1533546 - \Delta\theta_{\mathrm{B}}^{(0)} \cdot (-0{,}3290735) \\ &= -0{,}1533546 - 0{,}0067858 \cdot (-0{,}3290735) \\ &= -0{,}1511216 \end{aligned}$$

$$\theta_{\mathrm{C}}^{(1)} = \theta_{\mathrm{C}}^{(0)} + \Delta\theta_{\mathrm{C}}^{(0)} = 0 + \Delta\theta_{\mathrm{C}}^{(0)} = -0{,}1511216 \text{ rad}$$

$$\theta_{\mathrm{B}}^{(1)} = \theta_{\mathrm{B}}^{(0)} + \Delta\theta_{\mathrm{B}}^{(0)} = 0 + \Delta\theta_{\mathrm{B}}^{(0)} = 0{,}0067858 \text{ rad}$$

Точно решение (e=0,001) после три итерации

$$\max\left(\left|\Delta P^{(3)}\right|, \left|\Delta Q^{(3)}\right|\right) = 5{,}602837 \cdot 10^{-06}$$

$$\underline{U}_{\mathrm{C}}^{(3)} = 0{,}9149080 \cdot \mathrm{e}^{-\mathrm{j}9{,}52^{\circ}} \qquad \underline{U}_{\mathrm{B}}^{(3)} = 1{,}03 \cdot \mathrm{e}^{+\mathrm{j}0{,}39^{\circ}} \qquad Q_{\mathrm{B}} = 0{,}1307233$$

Пример 4.5 Систем со PU јазли

Vlezni podatoci za grankite

	R	X	G'xE-06	B'xE-06	Prenosen odnos
A - B	0.000000	0.8000000E-01	0.000000	10000.00	1.00/ 1.00
B - C	0.000000	0.3200000	0.000000	40000.00	1.00/ 1.00
A - C	0.000000	0.1600000	0.000000	20000.00	1.00/ 1.00

Vlezni podatoci za jazlite

Jazol	P	Q	U	Theta	Tip	Ubaz
C	-1.500000	-1.000000	?	?	PQ	1.00
B	0.6000000	?	1.030000	?	PV	1.00
A	?	?	1.050000	0.000000	S	1.00

Parametri od pi-ekv. semi

3-2 Y,Y'/2(0.000000 j -12.50000)(0.000000 j 0.5000000E-02)

2-1 Y,Y'/2(0.000000 j -3.125000)(0.000000 j 0.2000000E-01)

3-1 Y,Y'/2(0.000000 j -6.250000)(0.000000 j 0.1000000E-01)

Formiranje na matricata Y so inspekcija na mrezata

Matrica G

	C	B	C
C	0.000000	0.000000	0.000000
B	0.000000	0.000000	0.000000
A	0.000000	0.000000	0.000000

Matrica B

	B	A	A
C	-9.345000	3.125000	6.250000
B	3.125000	-15.60000	12.50000
A	6.250000	12.50000	-18.73500

Пример 4.5 Систем со PU јазли

Jakobijan i (dP,dQ) Resenie (1 iter):

C	B	C	dP,dQ	d(th),d
C 9.781250	-3.218750	0.000000	-1.500000	C -0.1511216
B -3.218750	16.73750	0.000000	0.6000000	B 0.6785803E-02
C 0.000000	0.000000	8.908751	-0.5637503	C -0.6328051E-01

.
.

Naponi vo jazlite posle 3.0 iteracii; (dP,dQ)max= 0.4947186E-05 < 0.1000000E-02

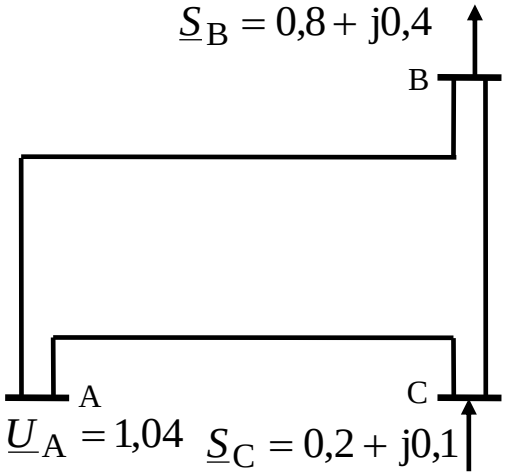
	(E +j F)	(U [ejth (rad.)])	(U[ejth])
C	(0.9023085 j-0.1513129)	(0.9149078 ej-0.1661494)	(0.91 ej -9.51966)
B	(1.029976 j 0.7085219E-02)	(1.030000 ej 0.6878908E-02)	(1.03 ej 0.39413)
A	(1.050000 j 0.000000)	(1.050000 ej 0.000000)	(1.05 ej 0.00000)

Zadadeni i presmetani moknosti vo jazlite (p.u.)

	(P +j Q)	(P +j Q)	(dP +j dQ)
C(PQ)	(-1.500000 j -1.000000)	(-1.499996 j-0.9999951)	(-0.3576279E-05 j-0.4947186E-05)
B(PV)	(0.6000000 j 0.000000)	(0.5999988 j 0.1307233)	(0.1251698E-05 j 0.000000)
A(S)	(0.000000 j 0.000000)	(0.8999976 j 1.215507)	(0.000000 j 0.000000)

Zagubi: (-0.5960464E-07 j 0.3462354)

Пример 4.6 Систем без PU јазли



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
A-B	3 – 1	0,24	0,32	0	0,0
B-C	1 – 2	0,12	0,16	0	0,0
A-C	3 – 2	0,06	0,08	0	0,0

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,04	0	S	3
B	–0,8	–0,4	?	?	PQ	1
C	0,2	0,1	?	?	PQ	2

$$\underline{Y}_{A-B} = \frac{1}{\underline{Z}_{A-B}} = \frac{1}{0{,}24 + j0{,}32} = 1{,}5 - j2$$

$$\underline{Y}_{C-A} = \frac{1}{\underline{Z}_{C-A}} = \frac{1}{0{,}06 + j0{,}08} = 6 - j8$$

$$\underline{Y}_{B-C} = \frac{1}{\underline{Z}_{B-C}} = \frac{1}{0{,}12 + j0{,}16} = 3 - j4$$

$$U_B^{(0)} = 1{,}0 \quad U_C^{(0)} = 1{,}0 \quad U_A = 1{,}04$$

$$\theta_B^{(0)} = 0 \quad \theta_C^{(0)} = 0 \quad \theta_A = 0$$

$$\underline{Y}_{AA} = \underline{Y}_{C-A} + \underline{Y}_{A-B} + \tfrac{1}{2} \cdot \left(\underline{Y}'_{C-A} + \underline{Y}'_{A-B} \right) = 7{,}5 - j9{,}97$$

$$\underline{Y}_{BB} = \underline{Y}_{B-C} + \underline{Y}_{A-B} + \tfrac{1}{2} \cdot \left(\underline{Y}'_{B-C} + \underline{Y}'_{A-B} \right) = 4{,}5 - j5{,}97$$

$$\underline{Y}_{CC} = \underline{Y}_{C-A} + \underline{Y}_{B-C} + \tfrac{1}{2} \cdot \left(\underline{Y}'_{C-A} + \underline{Y}'_{B-C} \right) = 9{,}0 - j11{,}96$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{B-C} = -(3{,}0 - j4{,}0) = -3{,}0 + j4{,}0$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = -\underline{Y}_{A-B} = -(1{,}5 - j2) = -1{,}5 + j2$$

$$\underline{Y}_{AC} = \underline{Y}_{CA} = -\underline{Y}_{A-C} = -(6{,}0 - j8{,}0) = -6{,}0 + j8{,}0$$

$n = 3 \quad \underline{\mathbf{Y}}_{3\times3}$

$$\underline{\mathbf{Y}} = \begin{bmatrix} 4{,}5 & -3{,}0 & -1{,}5 \\ -3{,}0 & 9{,}0 & -6{,}0 \\ -1{,}50 & -6{,}0 & 7{,}5 \end{bmatrix} + j \begin{bmatrix} -5{,}97 & 4{,}00 & 2{,}00 \\ 4{,}00 & -11{,}96 & 8{,}00 \\ 2{,}00 & 8{,}00 & -9{,}97 \end{bmatrix} \begin{matrix} B \\ C \\ A \end{matrix}$$

Пример 4.6 Систем без PU јазли

Решение со Њутн-Рафсонов метод

$$U_{\mathrm{B}}^{(0)} = 1{,}0 \quad U_{\mathrm{C}}^{(0)} = 1{,}0 \quad U_{\mathrm{A}} = 1{,}04$$
$$\theta_{\mathrm{B}}^{(0)} = 0 \quad \theta_{\mathrm{C}}^{(0)} = 0 \quad \theta_{\mathrm{A}} = 0$$

$$\underline{\boldsymbol{Y}} = \begin{bmatrix} 4{,}5 & -3{,}0 & -1{,}5 \\ -3{,}0 & 9{,}0 & -6{,}0 \\ -1{,}50 & -6{,}0 & 7{,}5 \end{bmatrix} + \mathrm{j}\begin{bmatrix} -5{,}97 & 4{,}00 & 2{,}00 \\ 4{,}00 & -11{,}96 & 8{,}00 \\ 2{,}00 & 8{,}00 & -9{,}97 \end{bmatrix} \begin{matrix} \mathrm{B} \\ \mathrm{C} \\ \mathrm{A} \end{matrix}$$

$$P_{\mathrm{B}_{\text{пресм.}}}^{(0)} = G_{\mathrm{BB}} \cdot U_{\mathrm{B}}^{(0)^2} + U_{\mathrm{B}}^{(0)} \cdot \left[U_{\mathrm{C}}^{(0)} \cdot \left(G_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{BC}} + B_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{BC}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{BA}} \cdot \cos\theta_{\mathrm{BA}} + B_{\mathrm{BA}} \cdot \sin\theta_{\mathrm{BA}} \right) \right]$$
$$= 4{,}5 \cdot 1{,}0^2 + 1{,}0 \cdot \left[1{,}0 \cdot \left(-3{,}0 \cdot 1 + 4{,}0 \cdot 0 \right) + 1{,}04 \cdot \left(-1{,}5 \cdot 1 + 2{,}0 \cdot 0 \right) \right] = -0{,}06$$

$$P_{\mathrm{C}_{\text{пресм.}}}^{(0)} = G_{\mathrm{CC}} \cdot U_{\mathrm{C}}^{(0)^2} + U_{\mathrm{C}}^{(0)} \cdot \left[U_{\mathrm{C}}^{(0)} \cdot \left(G_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}} + B_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{AC}} \cdot \cos\theta_{\mathrm{CA}} + B_{\mathrm{CA}} \cdot \sin\theta_{\mathrm{CA}} \right) \right]$$
$$= 9{,}0 \cdot 1{,}0^2 + 1{,}0 \cdot \left[1{,}0 \cdot \left(-3{,}0 \cdot 1 + 4{,}0 \cdot 0 \right) + 1{,}04 \cdot \left(-6{,}0 \cdot 0 + 8{,}0 \cdot 0 \right) \right] = -0{,}24$$

$$Q_{\mathrm{B}_{\text{пресм.}}}^{(0)} = -B_{\mathrm{BB}} \cdot U_{\mathrm{B}}^{(0)^2} + U_{\mathrm{B}}^{(0)} \cdot \left[U_{\mathrm{C}}^{(0)} \cdot \left(G_{\mathrm{BC}} \cdot \sin\theta_{\mathrm{BC}} - B_{\mathrm{BC}} \cdot \cos\theta_{\mathrm{BC}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{BA}} \cdot \sin\theta_{\mathrm{BA}} - B_{\mathrm{BA}} \cdot \cos\theta_{\mathrm{BA}} \right) \right]$$
$$= -\left(-5{,}97 \right) \cdot 1{,}0^2 + 1{,}0 \cdot \left[1{,}0 \cdot \left(-3{,}0 \cdot 0 - 4{,}0 \cdot 1 \right) + 1{,}04 \cdot \left(-1{,}5 \cdot 0 - 2{,}0 \cdot 1 \right) \right] = -0{,}11$$

$$Q_{\mathrm{C}_{\text{пресм.}}}^{(0)} = -B_{\mathrm{CC}} \cdot U_{\mathrm{C}}^{(0)^2} + U_{\mathrm{C}}^{(0)} \cdot \left[U_{\mathrm{B}}^{(0)} \cdot \left(G_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} - B_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}} \right) + U_{\mathrm{A}} \cdot \left(G_{\mathrm{CA}} \cdot \sin\theta_{\mathrm{CA}} - B_{\mathrm{CA}} \cdot \cos\theta_{\mathrm{CA}} \right) \right]$$
$$= -\left(-11{,}96 \right) \cdot 1{,}0^2 + 1{,}0 \cdot \left[1{,}0 \cdot \left(-3{,}0 \cdot 0 - 4{,}0 \cdot 1 \right) + 1{,}04 \cdot \left(-6{,}0 \cdot 0 - 8{,}0 \cdot 1 \right) \right] = -0{,}36$$

Пример 4.6 Систем без PU јазли

Решение со Њутн-Рафсонов метод

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,04	0	S	3
B	–0,8	–0,4	?	?	PQ	1
C	0,2	0,1	?	?	PQ	2

$$\begin{bmatrix} \boldsymbol{H} & \boldsymbol{N} \\ \boldsymbol{M} & \boldsymbol{L} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta} \\ \Delta\boldsymbol{U}/\boldsymbol{U} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P} \\ \Delta\boldsymbol{Q} \end{bmatrix}$$

$$\Delta P_{\text{B}}^{(0)} = P_{\text{B}_{\text{дадена}}} - P_{\text{B}_{\text{пресметана}}}^{(0)} = -0,8 - \left(-0,06\right) = -0,74$$

$$\Delta P_{\text{C}}^{(0)} = P_{\text{C}_{\text{дадена}}} - P_{\text{C}_{\text{пресметана}}}^{(0)} = 0,2 - \left(-0,24\right) = 0,44$$

$$\Delta Q_{\text{B}}^{(0)} = Q_{\text{B}_{\text{дадена}}} - Q_{\text{B}_{\text{пресметана}}}^{(0)} = -0,4 - \left(-0,11\right) = -0,299$$

$$\Delta Q_{\text{C}}^{(0)} = Q_{\text{C}_{\text{дадена}}} - Q_{\text{C}_{\text{пресметана}}}^{(0)} = 0,1 - \left(-0,36\right) = -0,46$$

$$n = 3 \rightarrow (n - 1 = 2)$$

$$q = 2$$

$$\begin{bmatrix} \boldsymbol{H}_{2\times2} & \boldsymbol{N}_{2\times2} \\ \boldsymbol{M}_{2\times2} & \boldsymbol{L}_{2\times2} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta}_{2\times1} \\ (\Delta\boldsymbol{U}/\boldsymbol{U})_{2\times1} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_{2\times1} \\ \Delta\boldsymbol{Q}_{2\times1} \end{bmatrix}$$

Пример 4.6 Систем без PU јазли **Решение со Њутн-Рафсонов метод**

$$H_{\mathrm{BC}} = L_{\mathrm{BC}} = U_{\mathrm{B}} \cdot U_{\mathrm{C}} \cdot (G_{\mathrm{BC}} \cdot \sin\theta_{\mathrm{BC}} - B_{\mathrm{BC}} \cdot \cos\theta_{\mathrm{BC}}) = 1{,}0 \cdot 1{,}0 \cdot (-3{,}0 \cdot 0 - 4{,}0 \cdot 1) = -4{,}0$$

$$H_{\mathrm{CB}} = L_{\mathrm{CB}} = U_{\mathrm{C}} \cdot U_{\mathrm{B}} \cdot (G_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}} - B_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}}) = 1{,}0 \cdot 1{,}0 \cdot (-3{,}0 \cdot 0 - 4{,}0 \cdot 1) = -4{,}0$$

$$H_{\mathrm{BB}} = -B_{\mathrm{BB}} \cdot U_{\mathrm{BB}}^2 - Q_{\mathrm{B}_{\text{пресметана}}} = -\left(-5{,}97\right) \cdot 1^2 - \left(-0{,}11\right) = 6{,}08$$

$$L_{\mathrm{BB}} = -B_{\mathrm{BB}} \cdot U_{\mathrm{BB}}^2 + Q_{\mathrm{B}_{\text{пресметана}}} = -\left(-5{,}97\right) \cdot 1^2 + \left(-0{,}11\right) = 5{,}86$$

$$H_{\mathrm{CC}} = -B_{\mathrm{CC}} \cdot U_{\mathrm{C}}^2 - Q_{\mathrm{C}_{\text{пресметана}}} = -\left(-11{,}96\right) \cdot 1^2 - \left(-0{,}36\right) = 12{,}32$$

$$L_{\mathrm{CC}} = -B_{\mathrm{CC}} \cdot U_{\mathrm{C}}^2 + Q_{\mathrm{C}_{\text{пресметана}}} = -\left(-11{,}96\right) \cdot 1^2 + \left(-0{,}36\right) = 11{,}6$$

$$N_{\mathrm{BB}} = G_{\mathrm{BB}} \cdot U_{\mathrm{B}}^2 + P_{\mathrm{B}_{\text{пресметана}}} = 4{,}5 \cdot 1^2 + \left(-0{,}06\right) = 4{,}44$$

$$M_{\mathrm{BB}} = -G_{\mathrm{BB}} \cdot U_{\mathrm{B}}^2 + P_{\mathrm{B}_{\text{пресметана}}} = -4{,}5 \cdot 1^2 + \left(-0{,}06\right) = -4{,}56$$

$$N_{\mathrm{CC}} = G_{\mathrm{CC}} \cdot U_{\mathrm{C}}^2 + P_{\mathrm{C}_{\text{пресметана}}} = 9{,}0 \cdot 1^2 + \left(-0{,}24\right) = 8{,}76$$

$$M_{\mathrm{CC}} = -G_{\mathrm{CC}} \cdot U_{\mathrm{C}}^2 + P_{\mathrm{C}_{\text{пресметана}}} = -9{,}0 \cdot 1^2 + \left(-0{,}24\right) = -9{,}24$$

$$N_{\mathrm{BC}} = -M_{\mathrm{BC}} = U_{\mathrm{B}} \cdot U_{\mathrm{C}} \cdot (G_{\mathrm{BC}} \cdot \cos\theta_{\mathrm{BC}} + B_{\mathrm{BC}} \cdot \sin\theta_{\mathrm{BC}}) = 1 \cdot 1 \cdot (-3{,}0 \cdot 1 + 4{,}0 \cdot 0) = -3{,}0$$

$$N_{\mathrm{CB}} = -M_{\mathrm{CB}} = U_{\mathrm{C}} \cdot U_{\mathrm{B}} \cdot (G_{\mathrm{CB}} \cdot \cos\theta_{\mathrm{CB}} + B_{\mathrm{CB}} \cdot \sin\theta_{\mathrm{CB}}) = 1 \cdot 1 \cdot (-3{,}0 \cdot 1 + 4{,}0 \cdot 0) = -3{,}0$$

Пример 4.6 Систем без PU јазли

Решение со Њутн-Рафсонов метод

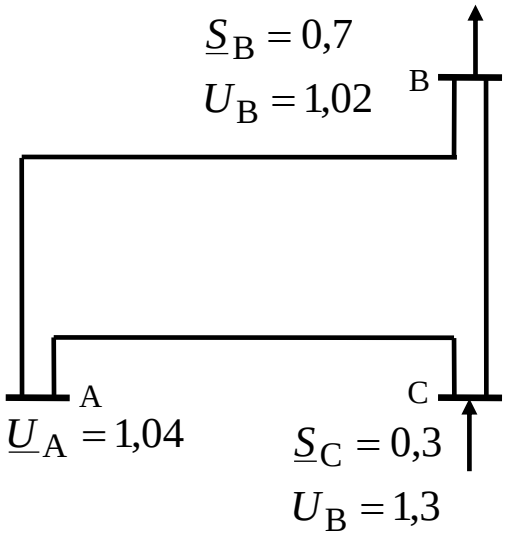
$$\left[\begin{array}{cc:cc} 6{,}08 & -4{,}0 & 4{,}44 & -3{,}0 \\ -4{,}0 & 12{,}32 & -3{,}0 & 8{,}76 \\ \hdashline -4{,}56 & 3{,}0 & 5{,}86 & -4{,}0 \\ 3{,}0 & -9{,}24 & -4{,}0 & 11{,}6 \end{array}\right] \cdot \left[\begin{array}{c} \Delta\theta_{\mathrm{B}}^{(0)} \\ \Delta\theta_{\mathrm{C}}^{(0)} \\ \hdashline \left(\Delta U_{\mathrm{B}}/U_{\mathrm{B}}\right)^{(0)} \\ \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \end{array}\right] = \left[\begin{array}{c} -0{,}74 \\ 0{,}44 \\ \hdashline -0{,}29 \\ 0{,}46 \end{array}\right]$$

$$\left[\begin{array}{c} \Delta\theta_{\mathrm{B}}^{(0)} \\ \Delta\theta_{\mathrm{C}}^{(0)} \\ \hdashline \Delta U_{\mathrm{B}}^{(0)} \\ \Delta U_{\mathrm{C}}^{(0)} \end{array}\right] = \left[\begin{array}{c} -0{,}06536043 \\ 0{,}01633411 \\ \hdashline -0{,}08142716 \\ 0{,}01546949 \end{array}\right]$$

$$\left[\begin{array}{c} \theta_{\mathrm{B}}^{(1)} \\ \theta_{\mathrm{C}}^{(1)} \end{array}\right] = \left[\begin{array}{c} -0{,}06536043 \\ 0{,}01633411 \end{array}\right]$$

$$\left[\begin{array}{c} U_{\mathrm{B}}^{(1)} \\ U_{\mathrm{C}}^{(1)} \end{array}\right] = \left[\begin{array}{c} 0{,}9185728 \\ 1{,}015470 \end{array}\right]$$

Пример 4.7 Систем со PU јазли



Податоци за јазлите (per unit)

Јазли	P	Q	U	θ	Тип	Р. бр
A	?	?	1,04	0	S	3
B	–0,7	?	1,02	?	PU	1
C	0,3	?	1,03	?	PU	2

$$n = 3 \Rightarrow \underline{\boldsymbol{Y}}_{3\times 3}$$

$$n-1 = 2;\ q = 0 \Rightarrow \boldsymbol{H}_{2\times 2}$$

$$n-1 = 2;\ q = 0 \Rightarrow \boldsymbol{L} = \boldsymbol{M} = \boldsymbol{N} = 0$$

Податоци за гранките (per unit)

Гранка	R	X	G'	B'
A-B	0,24	0,32	0	0,02
B-C	0,12	0,16	0	0,04
A-C	0,06	0,08	0	0,04

Решение со Њутн-Рафсонов метод

$$\begin{bmatrix} 6{,}324 & -4{,}2024 \\ -4{,}2024 & 12{,}772 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{B}}^{(0)} \\ \Delta\theta_{\mathrm{C}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}7-(-0{,}06119964) \\ 0{,}3-(-0{,}03089943) \end{bmatrix} = \begin{bmatrix} -0{,}6388004 \\ 0{,}3308994 \end{bmatrix} \qquad \begin{bmatrix} \underline{U}_{\mathrm{B}}^{(1)} \\ \underline{U}_{\mathrm{C}}^{(1)} \end{bmatrix} = \begin{bmatrix} 1{,}02 \cdot \mathrm{e}^{-\mathrm{j}0{,}1072443} \\ 1{,}03 \cdot \mathrm{e}^{-\mathrm{j}0{,}009378656} \end{bmatrix}$$

Пример 4.7 Систем со PU јазли

Vlezni podatoci za grankite

	R	X	G'xE-06	B'xE-06	Prenosen odnos
A - C	0.6000000E-01	0.8000000E-01	0.000000	40000.00	1.00/ 1.00
C - B	0.1200000	0.1600000	0.000000	40000.00	1.00/ 1.00
A - B	0.2400000	0.3200000	0.000000	20000.00	1.00/ 1.00

Vlezni podatoci za jazlite

Jazol	P	Q	U	Theta	Tip	Ubaz
B	-0.7000000	?	1.020000	?	PV	1.00
C	0.3000000	?	1.030000	?	PV	1.00
A	?	?	1.040000	0.000000	S	1.00

Parametri od pi-ekv. semi

3-2 Y,Y'/2(6.000000 j -8.000000)(0.000000 j 0.2000000E-01)

2-1 Y,Y'/2(3.000000 j -4.000000)(0.000000 j 0.2000000E-01)

3-1 Y,Y'/2(1.500000 j -2.000000)(0.000000 j 0.1000000E-01)

Formiranje na matricata Y so inspekcija na mrezata

Matrica G

	B	C	A
B	4.500000	-3.000000	-1.500000
C	-3.000000	9.000000	-6.000000
A	-1.500000	-6.000000	7.500000

Matrica B

	B	C	A
B	-5.970000	4.000000	2.000000
C	4.000000	-11.96000	8.000000
A	2.000000	8.000000	-9.970000

Пример 4.7 Систем со PU јазли

Jakobijan i (dP,dQ) — Resenie 1 iter:

B	C	dP,dQ	d(th),d
B 6.324000	-4.202400	-0.6388004	B -0.1072444
C -4.202400	12.77200	0.3308994	C -0.9378655E-02

.
.
.

Naponi vo jazlite posle 3.0 iteracii; (dP,dQ)max= 0.7748604E-06 < 0.1000000E-02

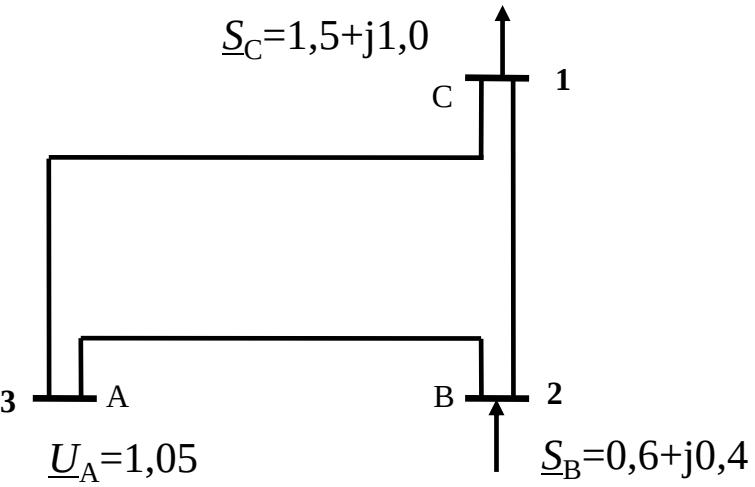
	(E +j F)	(U [ejth (rad.)])	(U[ejth])
B	(1.013391 j-0.1159241)	(1.020000 ej-0.1138972)	(1.02 ej -6.52583)
C	(1.029915 j-0.1319624E-01)	(1.030000 ej-0.1281224E-01)	(1.03 ej -0.73409)
A	(1.040000 j 0.000000)	(1.040000 ej 0.000000)	(1.04 ej 0.00000)

Zadadeni i presmetani moknosti vo jazlite (p.u.)

	(P +j Q)	(P +j Q)	(dP +j dQ)
B(PV)	(-0.7000000 j 0.000000)	(-0.6999992 j 0.4212851)	(-0.7748604E-06 j 0.000000)
C(PV)	(0.3000000 j 0.000000)	(0.2999998 j-0.2971937)	(0.2086163E-06 j 0.000000)
A(S)	(0.000000 j 0.000000)	(0.4553522 j-0.1563840)	(0.000000 j 0.000000)

Zagubi: (0.5535278E-01 j-0.3229260E-01)

FDC – XB – Пример 4.4 систем без PU јазли



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0	0,32	0	0,04
A-C	1-3	0	0,16	0	0,02
A-B	2-3	0	0,08	0	0,01

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,05	0	S	3
C	–1,5	–1,0	?	?	PQ	1
B	0,6	0,4	?	?	PQ	2

$U_C^{(0)} = 1{,}0 \quad U_B^{(0)} = 1{,}0 \quad U_A = 1{,}05$

$\theta_B^{(0)} = 0 \qquad \theta_A = 0$

$\theta_C^{(0)} = 0$

$n-1=2;\ q=2$

$\boldsymbol{B}'_{2\times 2},\quad \boldsymbol{B}''_{2\times 2}$

$$\underline{\boldsymbol{Y}} = \mathrm{j}\begin{bmatrix} -9{,}345 & 3{,}125 & 6{,}250 \\ 3{,}125 & -15{,}600 & 12{,}500 \\ 6{,}250 & 12{,}500 & -18{,}735 \end{bmatrix} \begin{matrix} \mathrm{C} \\ \mathrm{B} \\ \mathrm{A} \end{matrix}$$

$$\begin{matrix} \mathrm{C} & \mathrm{B} & \mathrm{A} \end{matrix}$$

FDC – XB – Пример 4.4 систем без PU јазли

Матрица $\boldsymbol{B}'$

ги занемаруваме напречните гранки од π-еквивалентните шеми и активните отпорности на надолжните гранки

$$B'_{ij} = -\sum_{j\in\beta_i} \frac{1}{X_{i-j}} \;\; ; \quad i,j = 1,\cdots,n-1; \;\; i \neq j$$

$$B'_{ii} = \sum_{j\in\alpha_i} \frac{1}{X_{i-j}}; \quad i = 1,\cdots,n-1$$

$$B'_{\mathrm{CB}} = B'_{\mathrm{BC}} = -\frac{1}{X_{\mathrm{C-B}}} = -\frac{1}{0{,}32} = -3{,}125$$

$$B'_{\mathrm{CC}} = \left(\frac{1}{X_{\mathrm{C-B}}} + \frac{1}{X_{\mathrm{C-A}}}\right) = \left(\frac{1}{0{,}32} + \frac{1}{0{,}16}\right) = 9{,}375$$

$$\boldsymbol{B}' = \begin{array}{c} \\ \mathrm{C} \\ \mathrm{B} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{C} & \mathrm{B} \end{array} \\ \begin{bmatrix} 9{,}375 & -3{,}125 \\ -3{,}125 & 15{,}625 \end{bmatrix} \end{array}$$

$$B'_{\mathrm{BB}} = \left(\frac{1}{X_{\mathrm{C-B}}} + \frac{1}{X_{\mathrm{B-A}}}\right) = \left(\frac{1}{0{,}32} + \frac{1}{0{,}08}\right) = 15{,}625$$

FDC – XB – Пример 4.4 систем без PU јазли

Матрица ***B”*** – матрица ***B*** со спротивен знак

$$\boldsymbol{B''} = -\boldsymbol{B}_{2\times 2} = \begin{array}{c} \begin{matrix} \text{C} & \text{B} \end{matrix} \\ \begin{bmatrix} 9{,}345 & -3{,}125 \\ -3{,}125 & 15{,}600 \end{bmatrix} \begin{matrix} \text{C} \\ \text{B} \end{matrix} \end{array}$$

$$\boldsymbol{B'} = \begin{array}{c} \begin{matrix} \text{C} & \text{B} \end{matrix} \\ \begin{bmatrix} 9{,}375 & -3{,}125 \\ -3{,}125 & 15{,}625 \end{bmatrix} \begin{matrix} \text{C} \\ \text{B} \end{matrix} \end{array}$$

FDC – XB – Пример 4.4 систем без PU јазли

Полу-итерација за пресметка на фазните агли на напоните

Пресметка на инјектираните активни моќности

$$P_{i_{\text{пресметана}}}^{(0)} = G_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \cos\theta_{ik} + B_{ik} \cdot \sin\theta_{ik} \right)$$

$$\underline{\boldsymbol{Y}} = \mathrm{j}\boldsymbol{B} \text{ и } \theta_{ik} = \theta_i^{(0)} - \theta_k^{(0)} = -\theta_{ki} = 0 \rightarrow \begin{cases} P_{\mathrm{C}_{\text{пресметана}}}^{(0)} = 0 & \Delta P_{\mathrm{C}}^{(0)} = P_{\mathrm{C}_{\mathbf{dadena}}} - P_{\mathrm{C}_{\mathbf{presmetana}}}^{(0)} = -1{,}5 - 0 = -1{,}5 \\ P_{\mathrm{B}_{\text{пресметана}}}^{(0)} = 0 & \Delta P_{\mathrm{B}}^{(0)} = P_{\mathrm{B}_{\mathbf{dadena}}} - P_{\mathrm{B}_{\mathbf{presmetana}}}^{(0)} = 0{,}6 - 0 = 0{,}6 \end{cases}$$

$$\Delta \boldsymbol{P}/\boldsymbol{U} = \begin{bmatrix} \Delta P_{\mathrm{C}}^{(0)} / U_{\mathrm{C}}^{(0)} \\ \Delta P_{\mathrm{B}}^{(0)} / U_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -1{,}5/1{,}0 \\ 0{,}6/1{,}0 \end{bmatrix} = \begin{bmatrix} -1{,}5 \\ 0{,}6 \end{bmatrix}$$

FDC – XB – Пример 4.4 систем без PU јазли

Полу-итерација за пресметка на фазните агли на напоните

$$\boldsymbol{B}' = \begin{array}{c} \begin{matrix} \text{C} & \text{B} \end{matrix} \\ \begin{bmatrix} 9{,}375 & -3{,}125 \\ -3{,}125 & 15{,}625 \end{bmatrix} \begin{matrix} \text{C} \\ \text{B} \end{matrix} \end{array}$$

$$\begin{bmatrix} 9{,}375 & -3{,}125 \\ -3{,}125 & 15{,}625 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\text{C}}^{(0)} \\ \Delta\theta_{\text{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -1{,}5 \\ 0{,}6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0{,}3333333 \\ 0 & 14{,}58333 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\text{C}}^{(0)} \\ \Delta\theta_{\text{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}16 \\ 0{,}1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0{,}3333333 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\text{C}}^{(0)} \\ \Delta\theta_{\text{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}16 \\ 6{,}857146 \cdot 10^{-3} \end{bmatrix}$$

$$\begin{aligned} \Delta\theta_{\text{C}}^{(0)} &= -0{,}16 - \Delta\theta_{\text{B}}^{(0)} \cdot (-0{,}3333333) \\ &= -0{,}16 - 6{,}857146 \cdot 10^{-3} \cdot (-0{,}3333333) \\ &= -0{,}1577143 \end{aligned}$$

$$\theta_{\text{C}}^{(1)} = \theta_{\text{C}}^{(0)} + \Delta\theta_{\text{C}}^{(0)} = 0{,}0 - 0{,}1577143 = -0{,}1577143$$

$$\theta_{\text{B}}^{(1)} = \theta_{\text{B}}^{(0)} + \Delta\theta_{\text{B}}^{(0)} = 0{,}0 + 6{,}857146 \cdot 10^{-3} = 6{,}857146 \cdot 10^{-3}$$

FDC – XB – Пример 4.4 систем без PU јазли

Полу-итерација за пресметка на ефективните вредности на напоните

Пресметка на инјектираните реактивни моќности

$$Q_{i_{\text{пресм.}}}^{(0)} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik} \right)$$

$$Q_{\text{C}_{\text{пресм.}}}^{(0)} = -B_{\text{CC}} \cdot U_{\text{C}}^{(0)^2} + U_{\text{C}}^{(0)} \cdot \left[U_{\text{B}}^{(0)} \cdot \left(G_{\text{CB}} \cdot \sin\theta_{\text{CB}}^{(1)} - B_{\text{CB}} \cdot \cos\theta_{\text{CB}}^{(1)} \right) + U_{\text{A}} \cdot \left(G_{\text{CA}} \cdot \sin\theta_{\text{CA}}^{(1)} - B_{\text{CA}} \cdot \cos\theta_{\text{CA}}^{(1)} \right) \right]$$

$$\theta_{\text{CA}}^{(1)} = \theta_{\text{C}}^{(1)} - \theta_{\text{A}} = -0{,}1577143 - 0 = -0{,}1577143$$

$$\theta_{\text{CB}}^{(1)} = \theta_{\text{C}}^{(1)} - \theta_{\text{B}}^{(1)} = -0{,}1577143 - 6{,}857146 \cdot 10^{-3} = -0{,}1645714$$

$$Q_{\text{C}_{\text{пресм.}}}^{(0)} = -\left(-9{,}345\right) \cdot 1^2 + 1{,}0 \cdot \left\{ 1 \cdot \left[0 - 3{,}125 \cdot \cos\left(-0{,}1645714\right) \right] + 1{,}05 \cdot \left[0 - 6{,}25 \cdot \cos\left(-0{,}1577143\right) \right] \right\}$$
$$= -0{,}2188292$$

$$\Delta Q_{\text{C}}^{(0)} = Q_{\text{C}_{\text{дадена}}} - Q_{\text{C}_{\text{пресметана}}}^{(0)} = -1{,}0 - \left(-0{,}2188292\right) = -0{,}7811708$$

FDC – XB – Пример 4.4 систем без PU јазли

Полу-итерација за пресметка на ефективните вредности на напоните

Пресметка на инјектираните реактивни моќности

$$Q_{i_{\text{пресм.}}}^{(0)} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik} \right)$$

$$Q_{\text{B}_{\text{пресм.}}}^{(0)} = -B_{\text{BB}} \cdot U_{\text{B}}^{(0)^2} + U_{\text{B}}^{(0)} \cdot \left[U_{\text{C}}^{(0)} \cdot \left(G_{\text{BC}} \cdot \sin\theta_{\text{BC}}^{(1)} - B_{\text{BC}} \cdot \cos\theta_{\text{BC}}^{(1)} \right) + U_{\text{A}} \cdot \left(G_{\text{BA}} \cdot \sin\theta_{\text{BA}}^{(1)} - B_{\text{BA}} \cdot \cos\theta_{\text{BA}}^{(1)} \right) \right]$$

$$\theta_{\text{BC}}^{(1)} = \theta_{\text{B}}^{(1)} - \theta_{\text{C}}^{(1)} = -\theta_{\text{CB}}^{(1)} = 0{,}1645714$$

$$\theta_{\text{BA}}^{(1)} = \theta_{\text{B}}^{(1)} - \theta_{\text{A}} = 6{,}857146 \cdot 10^{-3} - 0{,}0 = 6{,}857146 \cdot 10^{-3}$$

$$Q_{\text{B}_{\text{пресм.}}}^{(0)} = -\left(-15{,}6\right) \cdot 1{,}0^2 + 1{,}0 \cdot \left\{ \begin{array}{l} 1{,}0 \cdot \left[0 - 3{,}125 \cdot \cos\left(0{,}1645714\right) \right] + \\ 1{,}05 \cdot \left[0 - 12{,}5 \cdot \cos\left(6{,}857146 \cdot 10^{-3}\right) \right] \end{array} \right\}$$

$$= -0{,}6074686$$

$$\Delta Q_{\text{B}}^{(0)} = Q_{\text{B}_{\text{зададена}}} - Q_{\text{B}_{\text{пресметана}}}^{(0)} = 0{,}4 - \left(-0{,}6074686\right) = 1{,}007469$$

$$\Delta \boldsymbol{Q}/\boldsymbol{U} = \begin{bmatrix} \Delta Q_{\text{C}}^{(0)} / U_{\text{C}}^{(0)} \\ \Delta Q_{\text{B}}^{(0)} / U_{\text{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}7811708/1{,}0 \\ 1{,}007469/1{,}0 \end{bmatrix} = \begin{bmatrix} -0{,}7811708 \\ 1{,}007469 \end{bmatrix}$$

FDC – XB – Пример 4.4 систем без PU јазли

Полу-итерација за пресметка на ефективните вредности на напоните

$$\boldsymbol{B''} = \begin{array}{c} \\ \mathrm{C} \\ \mathrm{B} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{C} & \mathrm{B} \end{array} \\ \begin{bmatrix} 9{,}345 & -3{,}125 \\ -3{,}125 & 15{,}600 \end{bmatrix} \end{array}$$

$$\Delta \boldsymbol{Q}/\boldsymbol{U} = \begin{bmatrix} -0{,}7811708 \\ 1{,}007469 \end{bmatrix}$$

$$\begin{bmatrix} 9{,}345 & -3{,}125 \\ -3{,}125 & 15{,}600 \end{bmatrix} \cdot \begin{bmatrix} \Delta U_{\mathrm{C}}^{(0)} \\ \Delta U_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}7811708 \\ 1{,}007469 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0{,}3344035 \\ 0 & 14{,}55499 \end{bmatrix} \cdot \begin{bmatrix} \Delta U_{\mathrm{C}}^{(0)} \\ \Delta U_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -8{,}359239 \cdot 10^{-2} \\ 0{,}7462423 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0{,}3344035 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta U_{\mathrm{C}}^{(0)} \\ \Delta U_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -8{,}359239 \cdot 10^{-2} \\ 5{,}127056 \cdot 10^{-2} \end{bmatrix}$$

$$\begin{aligned} \Delta U_{\mathrm{C}}^{(0)} &= -8{,}359239 \cdot 10^{-2} - \Delta U_{\mathrm{B}}^{(0)} \cdot (-0{,}3344035) \\ &= -8{,}359239 \cdot 10^{-2} - \left(5{,}127056 \cdot 10^{-2}\right) \cdot (-0{,}3344035) \\ &= -6{,}644734 \cdot 10^{-2} \end{aligned}$$

FDC – XB – Пример 4.4 систем без PU јазли

Полу-итерација за пресметка на ефективните вредности на напоните

$$\begin{bmatrix} \Delta U_{\mathrm{C}}^{(0)} \\ \Delta U_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -6,644734 \cdot 10^{-2} \\ 5,127056 \cdot 10^{-2} \end{bmatrix}$$

$$U_{\mathrm{C}}^{(1)} = U_{\mathrm{C}}^{(0)} + \Delta U_{\mathrm{C}}^{(0)} = 1,0 - 6,644734 \cdot 10^{-2} = 0,9335527$$

$$U_{\mathrm{B}}^{(1)} = U_{\mathrm{B}}^{(0)} + \Delta U_{\mathrm{B}}^{(0)} = 1,0 + 5,127056 \cdot 10^{-2} = 1,051271$$

$$\begin{bmatrix} \theta_{\mathrm{C}}^{(0)} \\ \theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0,1577143 \\ 6,857146 \cdot 10^{-3} \end{bmatrix}$$

$$\underline{U}_{\mathrm{C}}^{(1)} = 0,9335527 \cdot \mathrm{e}^{-\mathrm{j}0,1577143}$$

$$\underline{U}_{\mathrm{B}}^{(1)} = 1,051271 \cdot \mathrm{e}^{-\mathrm{j}6,857146 \cdot 10^{-3}}$$

„Точно“ решение ($\varepsilon=10^{-3}$) после четири итерации

$$\underline{U}_{\mathrm{C}} = 0,9221520 \cdot \mathrm{e}^{-\mathrm{j}9,4^{\circ}}$$

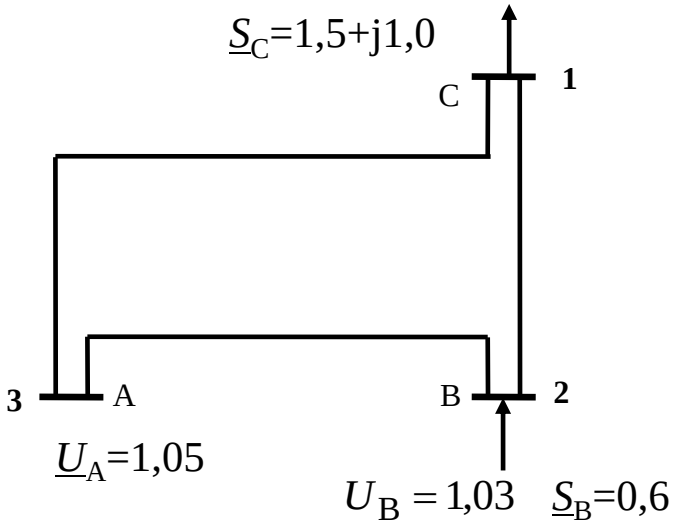
$$\underline{U}_{\mathrm{B}} = 1,047850 \cdot \mathrm{e}^{+\mathrm{j}0,37^{\circ}}$$

Решение со Њутн-Рафсонов метод

$$\underline{U}_{\mathrm{C}}^{(3)} = 0,9221041 \cdot \mathrm{e}^{-\mathrm{j}9,4^{\circ}}$$

$$\underline{U}_{\mathrm{B}}^{(3)} = 1,04784 \cdot \mathrm{e}^{+\mathrm{j}0,37^{\circ}}$$

FDC – XB – Пример 4.5 систем со PU јазли



Податоци за гранките (per unit)

Гранка		R	X	G'	B'
B-C	1-2	0	0,32	0	0,04
A-C	1-3	0	0,16	0	0,02
A-B	2-3	0	0,08	0	0,01

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,05	0	S	3
C	–1,5	–1,0	?	?	PQ	1
B	0,6	?	1,03	?	PU	2

$U_C^{(0)} = 1{,}0 \quad U_B = 1{,}03 \quad U_A = 1{,}05$

$\theta_C^{(0)} = 0 \quad \theta_B^{(0)} = 0 \quad \theta_A = 0$

$n-1=2;\ q=1 \quad \boldsymbol{B}'_{2\times 2},\ \boldsymbol{B}''_{1\times 1}$

$$\underline{\boldsymbol{Y}} = \mathrm{j}\begin{matrix}\\ \mathrm{C}\\ \mathrm{B}\\ \mathrm{A}\end{matrix}\begin{matrix}\begin{matrix}\mathrm{C} & \mathrm{B} & \mathrm{A}\end{matrix}\\ \begin{bmatrix} -9{,}345 & 3{,}125 & 6{,}250 \\ 3{,}125 & -15{,}600 & 12{,}500 \\ 6{,}250 & 12{,}500 & -18{,}735 \end{bmatrix}\end{matrix}$$

FDC – XB – Пример 4.5 систем со PU јазли

$$\underline{\underline{Y}} = \begin{array}{c} \\ \text{j} \end{array} \begin{array}{c} \begin{array}{ccc} \text{C} & \text{B} & \text{A} \end{array} \\ \begin{bmatrix} -9{,}345 & 3{,}125 & 6{,}250 \\ 3{,}125 & -15{,}600 & 12{,}500 \\ 6{,}250 & 12{,}500 & -18{,}735 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{C} \\ \text{B} \\ \text{A} \end{array}$$

Матрица B' – ги занемаруваме напречните гранки и активните отпорност на редните гранки

$$B'_{ij} = -\sum_{j \in \beta_i} \frac{1}{X_{i-j}} \;; \quad i, j = 1, \cdots, n-1; \;\; i \neq j$$

$$B'_{ii} = \sum_{j \in \alpha_i} \frac{1}{X_{i-j}}; \quad i = 1, \cdots, n-1$$

$$\boldsymbol{B'} = \begin{array}{c} \begin{array}{cc} \text{C} & \text{B} \end{array} \\ \begin{bmatrix} 9{,}375 & -3{,}125 \\ -3{,}125 & 15{,}625 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{C} \\ \text{B} \end{array}$$

$$B'_{\text{CB}} = B'_{\text{BC}} = -\frac{1}{X_{\text{C-B}}} = -\frac{1}{0{,}32} = -3{,}125$$

$$B'_{\text{CC}} = \left(\frac{1}{X_{\text{C-B}}} + \frac{1}{X_{\text{C-A}}} \right) = \left(\frac{1}{0{,}32} + \frac{1}{0{,}16} \right) = 9{,}375$$

$$B'_{\text{BB}} = \left(\frac{1}{X_{\text{C-B}}} + \frac{1}{X_{\text{B-A}}} \right) = \left(\frac{1}{0{,}32} + \frac{1}{0{,}08} \right) = 15{,}625$$

$$\boldsymbol{B''} = -\boldsymbol{B}_{1\times 1} = \begin{array}{c} \\ \begin{bmatrix} 9{,}345 \end{bmatrix} \end{array} \begin{array}{c} \text{C} \\ \text{C} \end{array}$$

FDC – XB – Пример 4.5 систем со PU јазли

Полу-итерација за пресметка на фазните агли на напоните

$$P_{i_{\text{пресметана}}}^{(0)} = G_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \cos\theta_{ik} + B_{ik} \cdot \sin\theta_{ik} \right)$$

$$\underline{\boldsymbol{Y}} = \mathrm{j}\boldsymbol{B} \;\mathtt{i}\; \theta_{ik} = \theta_i^{(0)} - \theta_k^{(0)} = -\theta_{ki} = 0 \rightarrow \begin{cases} P_{\mathrm{C}_{\text{пресметана}}}^{(0)} = 0 & \Delta P_{\mathrm{C}}^{(0)} = P_{\mathrm{C}_{\text{дадена}}} - P_{\mathrm{C}_{\text{пресметана}}}^{(0)} = -1,5 - 0 = -1,5 \\ P_{\mathrm{B}_{\text{пресметана}}}^{(0)} = 0 & \Delta P_{\mathrm{B}}^{(0)} = P_{\mathrm{B}_{\text{дадена}}} - P_{\mathrm{B}_{\text{пресметана}}}^{(0)} = 0,6 - 0 = 0,6 \end{cases}$$

$$\Delta \boldsymbol{P}/\boldsymbol{U} = \begin{bmatrix} \Delta P_{\mathrm{C}}^{(0)} / U_{\mathrm{C}}^{(0)} \\ \Delta P_{\mathrm{B}}^{(0)} / U_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} -1,5/1,0 \\ 0,6/1,03 \end{bmatrix} = \begin{bmatrix} -1,5 \\ 0,5825243 \end{bmatrix}$$

FDC – XB – Пример 4.5 систем со PU јазли

Полу-итерација за пресметка на фазните агли на напоните

$$\Delta \boldsymbol{P}/\boldsymbol{U} = \begin{bmatrix} \Delta P_{\mathrm{C}}^{(0)} / U_{\mathrm{C}}^{(0)} \\ \Delta P_{\mathrm{B}}^{(0)} / U_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} -1,5/1,0 \\ 0,6/1,03 \end{bmatrix} = \begin{bmatrix} -1,5 \\ 0,5825243 \end{bmatrix}$$

$$\boldsymbol{B}' = \begin{array}{c} \\ \mathrm{C} \\ \mathrm{B} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{C} & \mathrm{B} \end{array} \\ \begin{bmatrix} 9,375 & -3,125 \\ -3,125 & 15,625 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 9,375 & -3,125 \\ -3,125 & 15,625 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -1,5 \\ 0,5825243 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0,3333333 \\ 0 & 14,58333 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0,16 \\ 0,0825243 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0,3333333 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_{\mathrm{C}}^{(0)} \\ \Delta\theta_{\mathrm{B}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0,16 \\ 5,65881 \cdot 10^{-3} \end{bmatrix}$$

$$\begin{aligned} \Delta\theta_{\mathrm{C}}^{(0)} &= -0,16 - \Delta\theta_{\mathrm{B}}^{(0)} \cdot (-0,3333333) \\ &= -0,16 - 5,65881 \cdot 10^{-3} \cdot (-0,3333333) \\ &= -0,1581137 \end{aligned}$$

$$\theta_{\mathrm{C}}^{(1)} = \theta_{\mathrm{C}}^{(0)} + \Delta\theta_{\mathrm{C}}^{(0)} = 0,0 - 0,1581137 = -0,1581137$$

$$\theta_{\mathrm{B}}^{(1)} = \theta_{\mathrm{B}}^{(0)} + \Delta\theta_{\mathrm{B}}^{(0)} = 0,0 + 5,65881 \cdot 10^{-3} = 5,65881 \cdot 10^{-3}$$

FDC – XB – Пример 4.5 систем со PU јазли

$$\begin{matrix} \text{C} \\ \boldsymbol{B''} = \left[9{,}345\right] \\ \text{C} \end{matrix}$$

Полу-итерација за пресметка на ефективните вредности на напоните

$$Q_{i_{\text{пресметана}}}^{(0)} = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{k \in \alpha_i} U_k \cdot \left(G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik} \right)$$

$$Q_{\text{C}_{\text{пресметана}}}^{(0)} = -B_{\text{CC}} \cdot U_{\text{C}}^{(0)^2} + U_{\text{C}}^{(0)} \cdot \left[U_{\text{B}} \cdot \left(G_{\text{CB}} \cdot \sin\theta_{\text{CB}}^{(1)} - B_{\text{CB}} \cdot \cos\theta_{\text{CB}}^{(1)} \right) + U_{\text{A}} \cdot \left(G_{\text{CA}} \cdot \sin\theta_{\text{CA}}^{(1)} - B_{\text{CA}} \cdot \cos\theta_{\text{CA}}^{(1)} \right) \right]$$

$$\theta_{\text{CB}}^{(1)} = \theta_{\text{C}}^{(1)} - \theta_{\text{B}}^{(1)} = -0{,}1581137 - 5{,}65881 \cdot 10^{-3} = -0{,}1637725$$

$$\theta_{\text{CA}}^{(1)} = \theta_{\text{C}}^{(1)} - \theta_{\text{A}} = -0{,}1581137 - 0 = -0{,}1581137$$

$$\begin{aligned} Q_{\text{C}_{\text{пресметана}}}^{(0)} &= -\left(-9{,}345\right) \cdot 1^2 + 1{,}0 \cdot \left\{ \begin{matrix} 1{,}03 \cdot \left[0 - 3{,}125 \cdot \cos\left(-0{,}1637725\right) \right] + \\ 1{,}05 \cdot \left[0 - 6{,}25 \cdot \cos\left(-0{,}1581137\right) \right] \end{matrix} \right\} \\ &= -0{,}3113203 \end{aligned}$$

$$\Delta Q_{\text{C}}^{(0)} = Q_{\text{C}_{\text{дадена}}} - Q_{\text{C}_{\text{пресметана}}}^{(0)} = -1{,}0 - \left(-0{,}3113203\right) = -0{,}6886797$$

FDC – XB – Пример 4.5 систем со PU јазли

Полу-итерација за пресметка на ефективните вредности на напоните

$$\Delta Q_{\mathrm{C}}^{(0)} = Q_{\mathrm{C}_{\text{дадена}}} - Q_{\mathrm{C}_{\text{пресметана}}}^{(0)} = -1{,}0 - \left(-0{,}3113203\right) = -0{,}6886797$$

$$\Delta \boldsymbol{Q}/\boldsymbol{U} = \left[\Delta Q_{\mathrm{C}}^{(0)} / U_{\mathrm{C}}^{(0)}\right] = \left[-0{,}6886797/1{,}0\right] = \left[-0{,}6886797\right]$$

C

$$\boldsymbol{B''} = \left[9{,}345\right]\mathrm{C}$$

$$9{,}345 \cdot \Delta U_{\mathrm{C}}^{(0)} = -0{,}6886797$$

$$\Delta U_{\mathrm{C}}^{(0)} = \frac{-0{,}6886797}{9{,}345} = -0{,}073695$$

$$U_{\mathrm{C}}^{(1)} = U_{\mathrm{C}}^{(0)} + \Delta U_{\mathrm{C}}^{(0)} = 1{,}0 - 0{,}073695 = 0{,}926305$$

$$\underline{U}_{\mathrm{C}}^{(1)} = 0{,}926305 \cdot \mathrm{e}^{-\mathrm{j}0{,}1581137} \qquad \underline{U}_{\mathrm{B}}^{(1)} = 1{,}03 \cdot \mathrm{e}^{-\mathrm{j}5{,}65881 \cdot 10^{-3}}$$

Точно решение ($\varepsilon=10^{-3}$) после четири итерации

Решение со Њутн-Рафсонов метод

$$\left|\Delta P^{(4)}\right|_{\max} = 3{,}246069 \cdot 10^{-4} \qquad \left|\Delta Q^{(4)}\right|_{\max} = 2{,}541542 \cdot 10^{-4}$$

$$\underline{U}_{\mathrm{C}} = 0{,}9149503 \cdot \mathrm{e}^{-\mathrm{j}9{,}52^{\circ}}$$

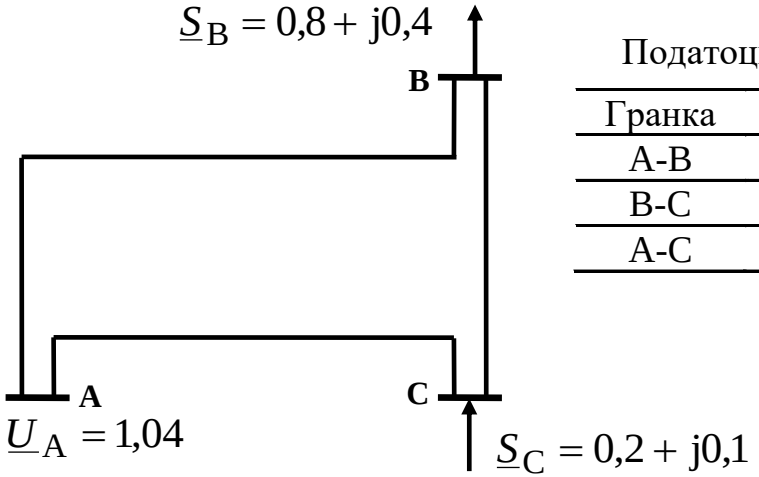
$$\underline{U}_{\mathrm{B}} = 1{,}03 \cdot \mathrm{e}^{+\mathrm{j}0{,}39^{\circ}}$$

$$Q_{\mathrm{B}} = 0{,}1305642$$

$$\underline{U}_{\mathrm{C}}^{(3)} = 0{,}9149080 \cdot \mathrm{e}^{-\mathrm{j}9{,}52^{\circ}}$$

$$\underline{U}_{\mathrm{B}}^{(3)} = 1{,}03 \cdot \mathrm{e}^{+\mathrm{j}0{,}39^{\circ}} \qquad Q_{\mathrm{B}} = 0{,}1307233$$

Пример 4.10 за систем без PU јазли



Податоци за гранките (per unit)

Гранка	R	X	G'	B'
A-B	0,24	0,32	0	0,02
B-C	0,12	0,16	0	0,04
A-C	0,06	0,08	0	0,04

Податоци за јазлите (per unit)

Јазол	P	Q	U	θ	Тип	Р. б.
A	?	?	1,04	0	S	3
B	–0,8	–0,4	?	?	PQ	1
C	0,2	0,1	?	?	PU	2

$n = 3 \quad \underline{\boldsymbol{Y}}_{3\times 3}$

$$U_B^{(0)} = 1{,}0 \quad U_C^{(0)} = 1{,}0 \quad U_A = 1{,}04$$

$$\theta_B^{(0)} = 0 \quad \theta_C^{(0)} = 0 \quad \theta_A = 0$$

$$\underline{Y}_{A-B} = \frac{1}{\underline{Z}_{A-B}} = \frac{1}{0{,}24 + j0{,}32} = 1{,}5 - j2$$

$$\underline{Y}_{AA} = \underline{Y}_{C-A} + \underline{Y}_{A-B} + \tfrac{1}{2}\cdot\left(\underline{Y}'_{C-A} + \underline{Y}'_{A-B}\right) = 7{,}5 - j9{,}97$$

$$\underline{Y}_{C-A} = \frac{1}{\underline{Z}_{C-A}} = \frac{1}{0{,}06 + j0{,}08} = 6 - j8$$

$$\underline{Y}_{BB} = \underline{Y}_{B-C} + \underline{Y}_{A-B} + \tfrac{1}{2}\cdot\left(\underline{Y}'_{B-C} + \underline{Y}'_{A-B}\right) = 4{,}5 - j5{,}97$$

$$\underline{Y}_{CC} = \underline{Y}_{C-A} + \underline{Y}_{B-C} + \tfrac{1}{2}\cdot\left(\underline{Y}'_{C-A} + \underline{Y}'_{B-C}\right) = 9{,}0 - j11{,}96$$

$$\underline{Y}_{B-C} = \frac{1}{\underline{Z}_{B-C}} = \frac{1}{0{,}12 + j0{,}16} = 3 - j4$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{B-C} = -(3{,}0 - j4{,}0) = -3{,}0 + j4{,}0$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = -\underline{Y}_{A-B} = -(1{,}5 - j2) = -1{,}5 + j2$$

$$\underline{Y}_{AC} = \underline{Y}_{CA} = -\underline{Y}_{A-C} = -(6{,}0 - j8{,}0) = -6{,}0 + j8{,}0$$

$$\underline{\boldsymbol{Y}} = \begin{bmatrix} 4{,}5 & -3{,}0 & -1{,}5 \\ -3{,}0 & 9{,}0 & -6{,}0 \\ -1{,}50 & -6{,}0 & 7{,}5 \end{bmatrix} + j\begin{bmatrix} -5{,}97 & 4{,}00 & 2{,}00 \\ 4{,}00 & -11{,}96 & 8{,}00 \\ 2{,}00 & 8{,}00 & -9{,}97 \end{bmatrix} \begin{matrix} B \\ C \\ A \end{matrix}$$

Пример 4.10 за систем без PU јазли **Решение со Њутн-Рафсонов метод**

$$P_{\text{B}_{\text{пресм.}}}^{(0)} = G_{\text{BB}} \cdot U_{\text{B}}^{(0)^2} + U_{\text{B}}^{(0)} \cdot \left[U_{\text{C}}^{(0)} \cdot \left(G_{\text{CB}} \cdot \cos\theta_{\text{BC}} + B_{\text{CB}} \cdot \sin\theta_{\text{BC}} \right) + U_{\text{A}} \cdot \left(G_{\text{BA}} \cdot \cos\theta_{\text{BA}} + B_{\text{BA}} \cdot \sin\theta_{\text{BA}} \right) \right]$$

$$= 4,5 \cdot 1,0^2 + 1,0 \cdot \left[1,0 \cdot \left(-3,0 \cdot 1 + 4,0 \cdot 0 \right) + 1,04 \cdot \left(-1,5 \cdot 1 + 2,0 \cdot 0 \right) \right] = -0,06$$

$$P_{\text{C}_{\text{пресм.}}}^{(0)} = G_{\text{CC}} \cdot U_{\text{C}}^{(0)^2} + U_{\text{C}}^{(0)} \cdot \left[U_{\text{C}}^{(0)} \cdot \left(G_{\text{CB}} \cdot \cos\theta_{\text{CB}} + B_{\text{CB}} \cdot \sin\theta_{\text{CB}} \right) + U_{\text{A}} \cdot \left(G_{\text{AC}} \cdot \cos\theta_{\text{CA}} + B_{\text{CA}} \cdot \sin\theta_{\text{CA}} \right) \right]$$

$$= 9,0 \cdot 1,0^2 + 1,0 \cdot \left[1,0 \cdot \left(-3,0 \cdot 1 + 4,0 \cdot 0 \right) + 1,04 \cdot \left(-6,0 \cdot 0 + 8,0 \cdot 0 \right) \right] = -0,24$$

$$Q_{\text{B}_{\text{пресм.}}}^{(0)} = -B_{\text{BB}} \cdot U_{\text{B}}^{(0)^2} + U_{\text{B}}^{(0)} \cdot \left[U_{\text{C}}^{(0)} \cdot \left(G_{\text{BC}} \cdot \sin\theta_{\text{BC}} - B_{\text{BC}} \cdot \cos\theta_{\text{BC}} \right) + U_{\text{A}} \cdot \left(G_{\text{BA}} \cdot \sin\theta_{\text{BA}} - B_{\text{BA}} \cdot \cos\theta_{\text{BA}} \right) \right]$$

$$= -\left(-5,97 \right) \cdot 1,0^2 + 1,0 \cdot \left[1,0 \cdot \left(-3,0 \cdot 0 - 4,0 \cdot 1 \right) + 1,04 \cdot \left(-1,5 \cdot 0 - 2,0 \cdot 1 \right) \right] = -0,11$$

$$Q_{\text{C}_{\text{пресм.}}}^{(0)} = -B_{\text{CC}} \cdot U_{\text{C}}^{(0)^2} + U_{\text{C}}^{(0)} \cdot \left[U_{\text{B}}^{(0)} \cdot \left(G_{\text{CB}} \cdot \sin\theta_{\text{CB}} - B_{\text{CB}} \cdot \cos\theta_{\text{CB}} \right) + U_{\text{A}} \cdot \left(G_{\text{CA}} \cdot \sin\theta_{\text{CA}} - B_{\text{CA}} \cdot \cos\theta_{\text{CA}} \right) \right]$$

$$= -\left(-11,96 \right) \cdot 1,0^2 + 1,0 \cdot \left[1,0 \cdot \left(-3,0 \cdot 0 - 4,0 \cdot 1 \right) + 1,04 \cdot \left(-6,0 \cdot 0 - 8,0 \cdot 1 \right) \right] = -0,36$$

$$\Delta P_{\text{B}}^{(0)} = P_{\text{B}_{\text{дадена}}} - P_{\text{B}_{\text{пресм.}}}^{(0)} = -0,8 - \left(-0,06 \right) = -0,74$$

$$\Delta P_{\text{C}}^{(0)} = P_{\text{C}_{\text{дадена}}} - P_{\text{C}_{\text{пресм.}}}^{(0)} = 0,2 - \left(-0,24 \right) = 0,44$$

$$\text{D}Q_{\text{B}}^{(0)} = Q_{\text{B}_{\text{дадена}}} - Q_{\text{B}_{\text{пресм.}}}^{(0)} = -0,4 - \left(-0,11 \right) = -0,299$$

$$\text{D}Q_{\text{C}}^{(0)} = Q_{\text{C}_{\text{дадена}}} - Q_{\text{C}_{\text{пресм.}}}^{(0)} = 0,1 - \left(-0,36 \right) = -0,46$$

$$\left[\begin{array}{c|c} \boldsymbol{H} & \boldsymbol{N} \\ \hline \boldsymbol{M} & \boldsymbol{L} \end{array} \right] \cdot \left[\begin{array}{c} \Delta\boldsymbol{\theta} \\ \hline \Delta\boldsymbol{U}/\boldsymbol{U} \end{array} \right] = \left[\begin{array}{c} \Delta\boldsymbol{P} \\ \hline \Delta\boldsymbol{Q} \end{array} \right]$$

$$n = 3 \rightarrow (n - 1 = 2)$$

$$q = 2$$

$$\begin{bmatrix} \boldsymbol{H}_{2\times 2} & \boldsymbol{N}_{2\times 2} \\ \boldsymbol{M}_{2\times 2} & \boldsymbol{L}_{2\times 2} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta}_{2\times 1} \\ (\Delta\boldsymbol{U}/\boldsymbol{U})_{2\times 1} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_{2\times 1} \\ \Delta\boldsymbol{Q}_{2\times 1} \end{bmatrix}$$

Пример 4.10 за систем без PU јазли **Решение со Њутн-Рафсонов метод**

$$H_{\text{BC}} = L_{\text{BC}} = U_{\text{B}} \cdot U_{\text{C}} \cdot (G_{\text{BC}} \cdot \sin\theta_{\text{BC}} - B_{\text{BC}} \cdot \cos\theta_{\text{BC}}) = 1{,}0 \cdot 1{,}0 \cdot (-3{,}0 \cdot 0 - 4{,}0 \cdot 1) = -4{,}0$$

$$H_{\text{CB}} = L_{\text{CB}} = U_{\text{C}} \cdot U_{\text{B}} \cdot (G_{\text{CB}} \cdot \sin\theta_{\text{CB}} - B_{\text{CB}} \cdot \cos\theta_{\text{CB}}) = 1{,}0 \cdot 1{,}0 \cdot (-3{,}0 \cdot 0 - 4{,}0 \cdot 1) = -4{,}0$$

$$H_{\text{BB}} = -B_{\text{BB}} \cdot U_{\text{BB}}^2 - Q_{\text{B}_{\text{пресм.}}} = -\left(-5{,}97\right) \cdot 1^2 - \left(-0{,}11\right) = 6{,}08$$

$$L_{\text{BB}} = -B_{\text{BB}} \cdot U_{\text{BB}}^2 + Q_{\text{B}_{\text{пресм.}}} = -\left(-5{,}97\right) \cdot 1^2 + \left(-0{,}11\right) = 5{,}86$$

$$H_{\text{CC}} = -B_{\text{CC}} \cdot U_{\text{C}}^2 - Q_{\text{C}_{\text{пресм.}}} = -\left(-11{,}96\right) \cdot 1^2 - \left(-0{,}36\right) = 12{,}32$$

$$L_{\text{CC}} = -B_{\text{CC}} \cdot U_{\text{C}}^2 + Q_{\text{C}_{\text{пресм.}}} = -\left(-11{,}96\right) \cdot 1^2 + \left(-0{,}36\right) = 11{,}6$$

$$N_{\text{BB}} = G_{\text{BB}} \cdot U_{\text{B}}^2 + P_{\text{B}_{\text{пресм.}}} = 4{,}5 \cdot 1^2 + \left(-0{,}06\right) = 4{,}44$$

$$M_{\text{BB}} = -G_{\text{BB}} \cdot U_{\text{B}}^2 + P_{\text{B}_{\text{пресм.}}} = -4{,}5 \cdot 1^2 + \left(-0{,}06\right) = -4{,}56$$

$$N_{\text{CC}} = G_{\text{CC}} \cdot U_{\text{C}}^2 + P_{\text{C}_{\text{пресм.}}} = 9{,}0 \cdot 1^2 + \left(-0{,}24\right) = 8{,}76$$

$$M_{\text{CC}} = -G_{\text{CC}} \cdot U_{\text{C}}^2 + P_{\text{C}_{\text{пресм.}}} = -9{,}0 \cdot 1^2 + \left(-0{,}24\right) = -9{,}24$$

$$N_{\text{BC}} = -M_{\text{BC}} = U_{\text{B}} \cdot U_{\text{C}} \cdot (G_{\text{BC}} \cdot \cos\theta_{\text{BC}} + B_{\text{BC}} \cdot \sin\theta_{\text{BC}}) = 1 \cdot 1 \cdot (-3{,}0 \cdot 1 + 4{,}0 \cdot 0) = -3{,}0$$

$$N_{\text{CB}} = -M_{\text{CB}} = U_{\text{C}} \cdot U_{\text{B}} \cdot (G_{\text{CB}} \cdot \cos\theta_{\text{CB}} + B_{\text{CB}} \cdot \sin\theta_{\text{CB}}) = 1 \cdot 1 \cdot (-3{,}0 \cdot 1 + 4{,}0 \cdot 0) = -3{,}0$$

Пример 4.10 за систем без PU јазли **Решение со Њутн-Рафсонов метод**

$$\left[\begin{array}{cc:cc} 6{,}08 & -4{,}0 & 4{,}44 & -3{,}0 \\ -4{,}0 & 12{,}32 & -3{,}0 & 8{,}76 \\ \hdashline -4{,}56 & 3{,}0 & 5{,}86 & -4{,}0 \\ 3{,}0 & -9{,}24 & -4{,}0 & 11{,}6 \end{array}\right] \cdot \left[\begin{array}{c} \Delta\theta_{\mathrm{B}}^{(0)} \\ \Delta\theta_{\mathrm{C}}^{(0)} \\ \hdashline \left(\Delta U_{\mathrm{B}}/U_{\mathrm{B}}\right)^{(0)} \\ \left(\Delta U_{\mathrm{C}}/U_{\mathrm{C}}\right)^{(0)} \end{array}\right] = \left[\begin{array}{c} -0{,}74 \\ 0{,}44 \\ \hdashline -0{,}29 \\ 0{,}46 \end{array}\right]$$

$$\left[\begin{array}{c} \Delta\theta_{\mathrm{B}}^{(0)} \\ \Delta\theta_{\mathrm{C}}^{(0)} \\ \hdashline \Delta U_{\mathrm{B}}^{(0)} \\ \Delta U_{\mathrm{C}}^{(0)} \end{array}\right] = \left[\begin{array}{c} -0{,}06536043 \\ 0{,}01633411 \\ \hdashline -0{,}08142716 \\ 0{,}01546949 \end{array}\right]$$

$$\left[\begin{array}{c} \theta_{\mathrm{B}}^{(1)} \\ \theta_{\mathrm{C}}^{(1)} \end{array}\right] = \left[\begin{array}{c} -0{,}06536043 \\ 0{,}01633411 \end{array}\right]$$

$$\left[\begin{array}{c} U_{\mathrm{B}}^{(1)} \\ U_{\mathrm{C}}^{(1)} \end{array}\right] = \left[\begin{array}{c} 0{,}9185728 \\ 1{,}015470 \end{array}\right]$$

Пример 4.10 за систем без PU јазли **Решение со FDC - XB**

Матрица B' – ги занемаруваме напречните гранки и активните отпорност на редните гранки

$$B'_{ij} = -\sum_{j\in\beta_i}\frac{1}{X_{i-j}}\ ;\quad i,j=1,\cdots,n-1;\ \ i\neq j$$

$$B'_{ii} = \sum_{j\in\alpha_i}\frac{1}{X_{i-j}};\quad i=1,\cdots,n-1$$

$$\boldsymbol{B}'_{2\times 2},\quad \boldsymbol{B}''_{2\times 2}$$

$$B'_{\mathrm{BB}} = \left(\frac{1}{X_{\mathrm{C-B}}}+\frac{1}{X_{\mathrm{B-A}}}\right) = \left(\frac{1}{0{,}16}+\frac{1}{0{,}32}\right) = 9{,}375$$

$$B'_{\mathrm{CC}} = \left(\frac{1}{X_{\mathrm{C-B}}}+\frac{1}{X_{\mathrm{C-A}}}\right) = \left(\frac{1}{0{,}16}+\frac{1}{0{,}08}\right) = 18{,}75$$

$$B'_{\mathrm{BC}} = B'_{\mathrm{CB}} = -\frac{1}{X_{\mathrm{C-B}}} = -\frac{1}{0{,}16} = -6{,}25$$

$$\boldsymbol{B}' = \begin{matrix}\\ \mathrm{B}\\ \mathrm{C}\end{matrix}\begin{matrix}\begin{matrix}\mathrm{B} & \mathrm{C}\end{matrix}\\ \begin{bmatrix} 9{,}375 & -6{,}25\\ -6{,}25 & 18{,}75\end{bmatrix}\end{matrix}$$

$$\Delta P_{\mathrm{B}}^{(0)} = P_{\mathrm{B}_{\text{дадена}}} - P_{\mathrm{B}_{\text{пресм.}}}^{(0)} = -0{,}8-\left(-0{,}06\right) = -0{,}74$$

$$\Delta P_{\mathrm{C}}^{(0)} = P_{\mathrm{C}_{\text{дадена}}} - P_{\mathrm{C}_{\text{пресм.}}}^{(0)} = 0{,}2-\left(-0{,}24\right) = 0{,}44$$

$$\Delta\boldsymbol{P}/\boldsymbol{U} = \begin{bmatrix}\Delta P_{\mathrm{B}}^{(0)}/U_{\mathrm{B}}^{(0)}\\ \Delta P_{\mathrm{C}}^{(0)}/U_{\mathrm{C}}^{(0)}\end{bmatrix} = \begin{bmatrix}-0{,}74/1{,}0\\ 0{,}44/1{,}0\end{bmatrix} = \begin{bmatrix}-0{,}74\\ 0{,}44\end{bmatrix}$$

$$\begin{bmatrix} 9{,}375 & -6{,}25\\ -6{,}25 & 18{,}75\end{bmatrix}\cdot\begin{bmatrix}\Delta\theta_{\mathrm{B}}^{(0)}\\ \Delta\theta_{\mathrm{C}}^{(0)}\end{bmatrix} = \begin{bmatrix}-0{,}74\\ 0{,}44\end{bmatrix}$$

$$\begin{bmatrix}\Delta\theta_{\mathrm{B}}^{(0)}\\ \Delta\theta_{\mathrm{C}}^{(0)}\end{bmatrix} = \begin{bmatrix}\theta_{\mathrm{B}}^{(0)}\\ \theta_{\mathrm{C}}^{(0)}\end{bmatrix} = \begin{bmatrix}-8{,}137145\cdot 10^{-2}\\ -3{,}657164\cdot 10^{-3}\end{bmatrix}$$

$$\left[\theta^{(1)}\right] = \left[\theta^{(0)}\right] + \left[\Delta\theta^{(0)}\right]$$

Пример 4.10 за систем без PU јазли **Решение со FDC - XB**

$$\boldsymbol{B''} = -\boldsymbol{B}_{2\times 2} = \begin{bmatrix} 5{,}97 & -4{,}00 \\ -4{,}00 & 11{,}96 \end{bmatrix} \begin{matrix} \text{B} \\ \text{C} \end{matrix}$$

$$\Delta Q_{\text{B}}^{(0)} = Q_{\text{B}_{\text{дадена}}} - Q_{\text{B}_{\text{пресм.}}}^{(0)} = -0,4 - \left(-0,2686629\right) = -0,6686629$$

$$\Delta Q_{\text{C}}^{(0)} = Q_{\text{C}_{\text{дадена}}} - Q_{\text{C}_{\text{пресм.}}}^{(0)} = 0,1 - \left(-0,5579596\right) = 0,6579596$$

$$\Delta \boldsymbol{Q}/\boldsymbol{U} = \begin{bmatrix} \Delta Q_{\text{B}}^{(0)} / U_{\text{B}}^{(0)} \\ \Delta Q_{\text{C}}^{(0)} / U_{\text{C}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0,6686629/1,0 \\ 0,6579596/1,0 \end{bmatrix} = \begin{bmatrix} -0,6686629 \\ 0,6579596 \end{bmatrix}$$

$$\begin{bmatrix} 5{,}97 & -4{,}00 \\ -4{,}00 & 11{,}96 \end{bmatrix} . \begin{bmatrix} \Delta U_{\text{B}}^{(0)} \\ \Delta U_{\text{C}}^{(0)} \end{bmatrix} = \begin{bmatrix} -0,6686629 \\ 0,6579596 \end{bmatrix}$$

$$\begin{bmatrix} U_{\text{B}}^{(1)} \\ U_{\text{C}}^{(1)} \end{bmatrix} = \begin{bmatrix} U_{\text{B}}^{(0)} \\ U_{\text{C}}^{(0)} \end{bmatrix} + \begin{bmatrix} \Delta U_{\text{B}}^{(0)} \\ \Delta U_{\text{C}}^{(0)} \end{bmatrix} = \begin{bmatrix} 1,0 - 0,0968457 \\ 1,0 + 0,0226235 \end{bmatrix} = \begin{bmatrix} 0,9031543 \\ 1,0226235 \end{bmatrix}$$

Пример 4.10 за систем без PU јазли

„Точно“ решение ($\varepsilon=10^{-3}$)

Њутн-Рафсон

$$\begin{bmatrix} \underline{U}_{\mathrm{B}}^{(3)} \\ \underline{U}_{\mathrm{C}}^{(3)} \end{bmatrix} = \begin{bmatrix} 0{,}9019585 \cdot \mathrm{e}^{-\mathrm{j}4{,}07^\circ} \\ 1{,}008814 \cdot \mathrm{e}^{-\mathrm{j}0{,}94^\circ} \end{bmatrix}$$

$$\begin{bmatrix} \underline{S}_{\mathrm{B}}^{(3)} \\ \underline{S}_{\mathrm{C}}^{(3)} \\ \underline{S}_{\mathrm{A}}^{(3)} \end{bmatrix} = \begin{bmatrix} -0{,}7999992 - \mathrm{j}0{,}3999994 \\ 0{,}1999992 + \mathrm{j}0{,}1000010 \\ 0{,}6856035 + \mathrm{j}0{,}3165739 \end{bmatrix}$$

$$\Delta\underline{S} = 0{,}0856035 + \mathrm{j}0{,}01657556$$

FDC - XB

$$\begin{bmatrix} \underline{U}_{\mathrm{B}}^{(4)} \\ \underline{U}_{\mathrm{C}}^{(4)} \end{bmatrix} = \begin{bmatrix} 0{,}9020121 \cdot \mathrm{e}^{-\mathrm{j}4{,}06^\circ} \\ 1{,}008852 \cdot \mathrm{e}^{-\mathrm{j}0{,}94^\circ} \end{bmatrix}$$

$$\begin{bmatrix} \underline{S}_{\mathrm{B}}^{(4)} \\ \underline{S}_{\mathrm{C}}^{(4)} \\ \underline{S}_{\mathrm{A}}^{(4)} \end{bmatrix} = \begin{bmatrix} -0{,}7995011 - \mathrm{j}0{,}4002431 \\ 0{,}2006007 + \mathrm{j}0{,}9991586 \\ 0{,}6844019 + \mathrm{j}0{,}3167574 \end{bmatrix}$$

$$\Delta\underline{S} = 0{,}08550143 + \mathrm{j}0{,}0164302$$

Пример 4.10 за систем без PU јазли Решение со приближен метод – DC Load Flow

$$\boldsymbol{B}'''_{2\times 2}$$

$$B'''_{ij} = -\sum_{j\in\beta_i}\frac{1}{X_{i-j}}\ ;\quad i,j=1,\cdots,n-1;\ \ i\neq j$$

$$B'''_{ii} = \sum_{j\in\alpha_i}\frac{1}{X_{i-j}};\quad i=1,\cdots,n-1$$

$$B'''_{\mathrm{BB}} = \left(\frac{1}{X_{\mathrm{C-B}}}+\frac{1}{X_{\mathrm{B-A}}}\right) = \left(\frac{1}{0{,}16}+\frac{1}{0{,}32}\right) = 9{,}375$$

$$B'''_{\mathrm{CC}} = \left(\frac{1}{X_{\mathrm{C-B}}}+\frac{1}{X_{\mathrm{C-A}}}\right) = \left(\frac{1}{0{,}16}+\frac{1}{0{,}08}\right) = 18{,}75$$

$$B'''_{\mathrm{BC}} = B'_{\mathrm{CB}} = -\frac{1}{X_{\mathrm{C-B}}} = -\frac{1}{0{,}16} = -6{,}25$$

$$\boldsymbol{B}''' = \begin{matrix}\\ \mathrm{B}\\ \mathrm{C}\end{matrix}\!\!\begin{matrix}\begin{matrix}\mathrm{B} & \mathrm{C}\end{matrix}\\ \begin{bmatrix}9{,}375 & -6{,}25\\ -6{,}25 & 18{,}75\end{bmatrix}\end{matrix}\qquad \boldsymbol{P} = \begin{bmatrix}-0{,}8\\ 0{,}2\end{bmatrix}$$

$$\begin{bmatrix}9{,}375 & -6{,}25\\ -6{,}25 & 18{,}75\end{bmatrix}\cdot\begin{bmatrix}\theta_{\mathrm{B}}\\ \theta_{\mathrm{C}}\end{bmatrix} = \begin{bmatrix}-0{,}8\\ 0{,}2\end{bmatrix}\qquad \begin{bmatrix}\theta_{\mathrm{B}}\\ \theta_{\mathrm{C}}\\ \theta_{\mathrm{A}}\end{bmatrix} = \begin{bmatrix}-0{,}08533333\\ -0{,}02285714\\ 0{,}0\end{bmatrix}$$

$$P_{\mathrm{A-B}} = \frac{\theta_{\mathrm{A}}-\theta_{\mathrm{B}}}{X_{\mathrm{A-B}}}\qquad P_{\mathrm{B-C}} = \frac{\theta_{\mathrm{B}}-\theta_{\mathrm{C}}}{X_{\mathrm{B-C}}}\qquad P_{\mathrm{A-C}} = \frac{\theta_{\mathrm{A}}-\theta_{\mathrm{C}}}{X_{\mathrm{A-C}}}$$

Пример 4.10 за систем без PU јазли Решение со приближен метод – DC Load Flow

Елемнет *i–k*	Активна моќност на почетокот (per unit)		Разлика (%)	Активна моќност на крајот (per unit)		Разлика (%)
	„Точна“	Приближна		„Точна“	Приближна	
A–C	0,3335541	0,2857143	14,3	–0,3260143	–0,2857143	12,4
C–B	0,5260140	0,4857143	7,7	–0,4836297	–0,4857143	–0,4
A–B	0,3520495	0,3142857	10,7	–0,3163692	–0,3142857	0,7

$$\underline{S}_{k-l} = P_{k-l} + \mathrm{j}Q_{k-l} = \underline{U}_k \cdot \underline{I}^*_{k-l} = \underline{U}_k \cdot \left[\left(\underline{U}^*_k - \underline{U}^*_l \right) \cdot \underline{Y}^*_{k-l} + \underline{U}^*_k \cdot \frac{\left(\underline{Y}'_{k-l} \right)^*}{2} \right]$$

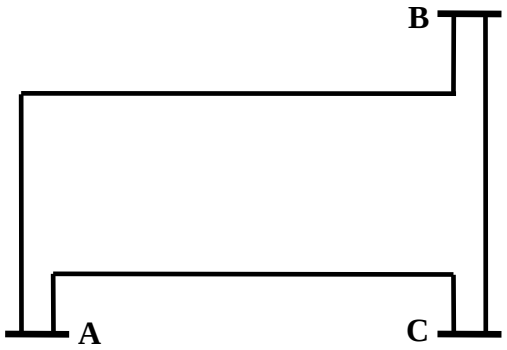
$$\underline{S}_{l-k} = P_{l-k} + \mathrm{j}Q_{l-k} = \underline{U}_l \cdot \underline{I}^*_{l-k} = \underline{U}_l \cdot \left[\left(\underline{U}^*_l - \underline{U}^*_k \right) \cdot \underline{Y}^*_{k-l} + \underline{U}^*_l \cdot \frac{\left(\underline{Y}'_{k-l} \right)^*}{2} \right]$$

$\Delta\underline{S} = 0{,}0856035 + \mathrm{j}0{,}01657556$

$\Delta\underline{S} = 0{,}08550143 + \mathrm{j}0{,}0164302$

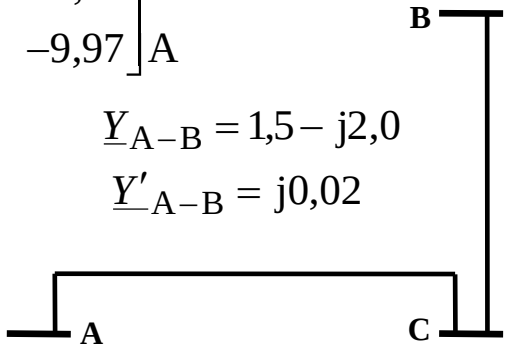
$$\Delta\underline{S} = 0{,}08560443 + \mathrm{j}0{,}01657708$$

Пример 4.10 за систем без PU јазли



Да се определи матрицата $\underline{Y}$ ако се исклучи гранката A–B

$$\underline{Y} = \begin{bmatrix} 4,5 & -3,0 & -1,5 \\ -3,0 & 9,0 & -6,0 \\ -1,50 & -6,0 & 7,5 \end{bmatrix} + \mathrm{j}\begin{bmatrix} -5,97 & 4,00 & 2,00 \\ 4,00 & -11,96 & 8,00 \\ 2,00 & 8,00 & -9,97 \end{bmatrix} \begin{matrix} \mathrm{B} \\ \mathrm{C} \\ \mathrm{A} \end{matrix}$$



$$\underline{Y}_{\mathrm{A-B}} = 1,5 - \mathrm{j}2,0$$

$$\underline{Y}'_{\mathrm{A-B}} = \mathrm{j}0,02$$

$$\underline{Y}_{\mathrm{AA}} = \underline{Y}_{\mathrm{AA}} - \underline{Y}_{\mathrm{A-B}} - \tfrac{1}{2}\cdot\underline{Y}'_{\mathrm{A-B}} = 7,5 - \mathrm{j}9,97 - 1,5 + \mathrm{j}2,0 - \mathrm{j}0,01 = 6,0 - \mathrm{j}7,98$$

$$\underline{Y}_{\mathrm{BB}} = \underline{Y}_{\mathrm{BB}} - \underline{Y}_{\mathrm{A-B}} - \tfrac{1}{2}\cdot\underline{Y}'_{\mathrm{A-B}} = 4,5 - \mathrm{j}5,97 - 1,5 + \mathrm{j}2,0 - \mathrm{j}0,01 = 3,0 - \mathrm{j}3,98$$

$$\underline{Y}_{\mathrm{AB}} = \underline{Y}_{\mathrm{BA}} = \underline{Y}_{\mathrm{AB}} + \underline{Y}_{\mathrm{A-B}} = -1,5 + \mathrm{j}2 + 1,5 - \mathrm{j}2 = 0$$

$$\underline{Y} = \begin{bmatrix} 3,0 & -3,0 & 0 \\ -3,0 & 9,0 & -6,0 \\ 0 & -6,0 & 6,0 \end{bmatrix} + \mathrm{j}\begin{bmatrix} -3,98 & 4,00 & 0 \\ 4,00 & -11,96 & 8,00 \\ 0 & 8,00 & -7,98 \end{bmatrix} \begin{matrix} \mathrm{B} \\ \mathrm{C} \\ \mathrm{A} \end{matrix}$$

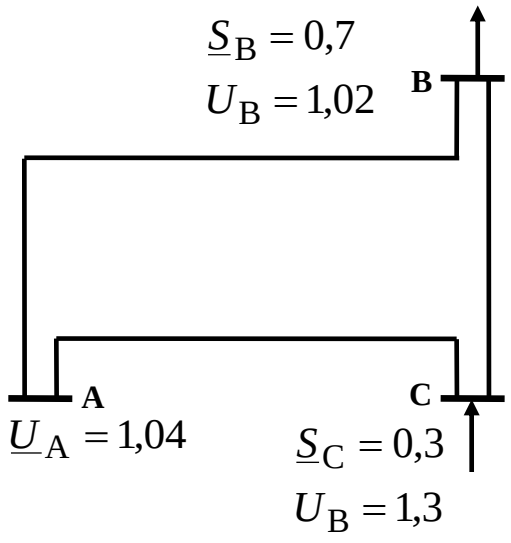
Од матрицата на адмитанции да се определат редните импеданции на гранките

$$\underline{Y}_{\mathrm{CB}} = \underline{Y}_{\mathrm{BC}} = -\underline{Y}_{\mathrm{B-C_{ekv.}}} = \frac{1}{\underline{Z}_{\mathrm{B-C_{ekv.}}}} = -3,0 + \mathrm{j}4,0 \Rightarrow \underline{Z}_{\mathrm{B-C_{ekv.}}} = \frac{1}{-3,0 + \mathrm{j}4,0} = 0,12 + \mathrm{j}0,16$$

$$\underline{Y}_{\mathrm{AC}} = \underline{Y}_{\mathrm{CA}} = -\underline{Y}_{\mathrm{A-C_{ekv.}}} = \frac{1}{\underline{Z}_{\mathrm{A-C_{ekv.}}}} = -6,0 + \mathrm{j}8,0 \Rightarrow \underline{Z}_{\mathrm{A-C_{ekv.}}} = \frac{1}{-6,0 + \mathrm{j}8,0} = 0,06 + \mathrm{j}0,08$$

$$\underline{Y}_{\mathrm{AB}} = -\underline{Y}_{\mathrm{A-B_{ekv.}}} = \frac{1}{\underline{Z}_{\mathrm{A-B_{ekv.}}}} = 0 + \mathrm{j}0 \Rightarrow \underline{Z}_{\mathrm{A-C_{ekv.}}} = \infty$$

Пример 4.11 – Систем со PU јазли



Податоци за гранките (per unit)

Гранка	R	X	G'	B'
A-B	0,24	0,32	0	0,02
B-C	0,12	0,16	0	0,04
A-C	0,06	0,08	0	0,04

Податоци за гранките (per unit)

Јазол	P	Q	U	θ	Тип	Р. бр.
A	?	?	1,04	0	S	3
B	–0,7	?	1,02	?	PQ	1
C	0,3	?	1,03	?	PQ	2

$$n = 3 \Rightarrow \boldsymbol{\underline{Y}}_{3\times 3}$$

$$n-1 = 2;\ q = 0 \Rightarrow \boldsymbol{H}_{2\times 2}$$

$$n-1 = 2;\ q = 0 \Rightarrow \boldsymbol{B}'_{2\times 2};\ \boldsymbol{B}'' = 0$$

Њутн-Рафсон

$$\begin{bmatrix} 6{,}324 & -4{,}2024 \\ -4{,}2024 & 12{,}772 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_B^{(0)} \\ \Delta\theta_C^{(0)} \end{bmatrix} = \begin{bmatrix} -0{,}7-(-0{,}06119964) \\ 0{,}3-(-0{,}03089943) \end{bmatrix} = \begin{bmatrix} -0{,}6388004 \\ 0{,}3308994 \end{bmatrix} \quad \begin{bmatrix} \underline{U}_B^{(1)} \\ \underline{U}_C^{(1)} \end{bmatrix} = \begin{bmatrix} 1{,}02\cdot e^{-j0{,}1072443} \\ 1{,}03\cdot e^{-j0{,}009378656} \end{bmatrix}$$

FDC - XB

$$\begin{bmatrix} 9{,}375 & -6{,}25 \\ -6{,}250 & 18{,}75 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_B^{(0)} \\ \Delta\theta_C^{(0)} \end{bmatrix} = \begin{bmatrix} (-0{,}7-(-0{,}06119964))/1{,}02 \\ (0{,}3-(-0{,}03089943))/1{,}03 \end{bmatrix} = \begin{bmatrix} -0{,}6262749 \\ 0{,}3212616 \end{bmatrix} \quad \begin{bmatrix} \underline{U}_B^{(1)} \\ \underline{U}_C^{(1)} \end{bmatrix} = \begin{bmatrix} 1{,}02\cdot e^{-j0{,}07120287} \\ 1{,}03\cdot e^{-j0{,}006600340} \end{bmatrix}$$