

# СВЕТЛЕЧКИ ОТСЕЧКИ – ЛИНИСКИ ИЗВОРИ

$$dE_T = I'_{\alpha\gamma} \frac{\cos \beta}{DT^2} dx = I'_{\alpha\gamma} \frac{\cos \beta}{r^2} dx$$

$$E_T = \int_0^d \frac{I'_{\alpha\gamma} \cdot \cos \beta}{r^2} \cdot dx$$

$$\cos \gamma = \frac{\overline{A_1 T}}{r} \Rightarrow r = \frac{h}{\cos \alpha \cdot \cos \gamma}$$

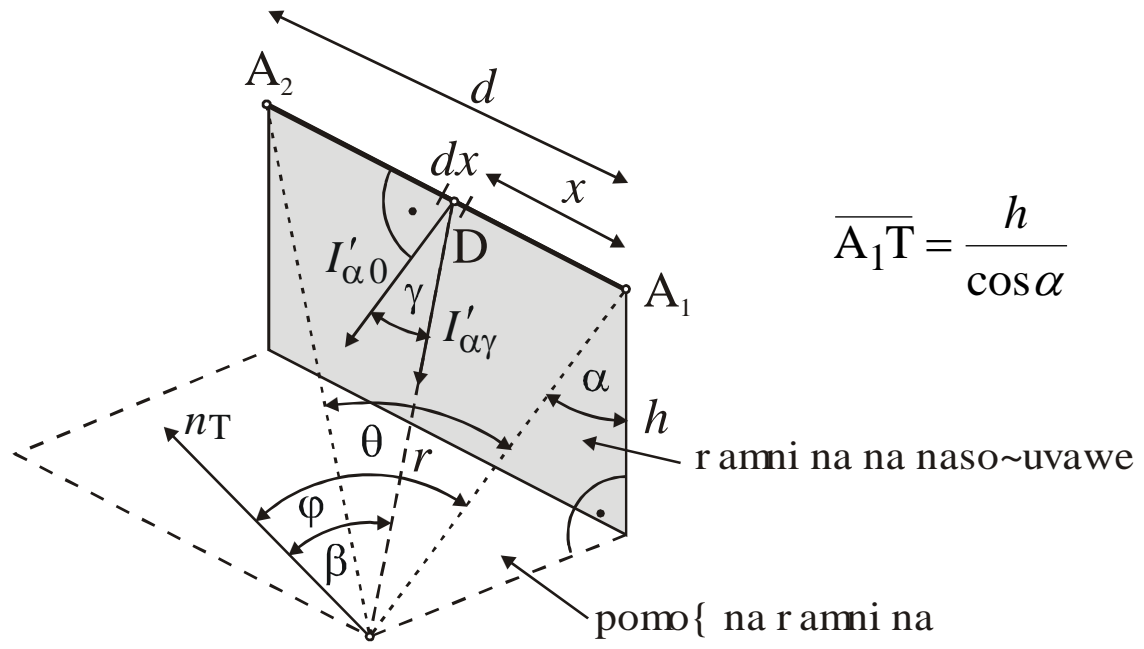
$$\tan \gamma = \frac{x}{\overline{A_1 T}} \Rightarrow x = \frac{h}{\cos \alpha} \cdot \tan \gamma$$

$$dx = \frac{h \cdot d\gamma}{\cos \alpha \cdot \cos^2 \gamma}$$

$$dE = \frac{I'_{\alpha\gamma}}{h} \cdot \cos \alpha \cdot \cos \beta \cdot d\gamma$$

$$\theta = \arctan\left(\frac{\overline{A_1 A_2}}{\overline{A_1 T}}\right) = \arctan\left(\frac{d}{\frac{h}{\cos \alpha}}\right) = \arctan\left(\frac{d}{h} \cdot \cos \alpha\right)$$

$$I'_{\alpha\gamma} = I'_{\alpha 0} \cdot \cos^s \gamma$$



$$\overline{A_1 T} = \frac{h}{\cos \alpha}$$

$$E_T = \frac{1}{h} \cdot \cos \alpha \int_0^\theta I'_{\alpha\gamma} \cdot \cos \beta \cdot d\gamma$$

$$E = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \int_0^\theta \cos \beta \cdot \cos^s \gamma \cdot d\gamma$$

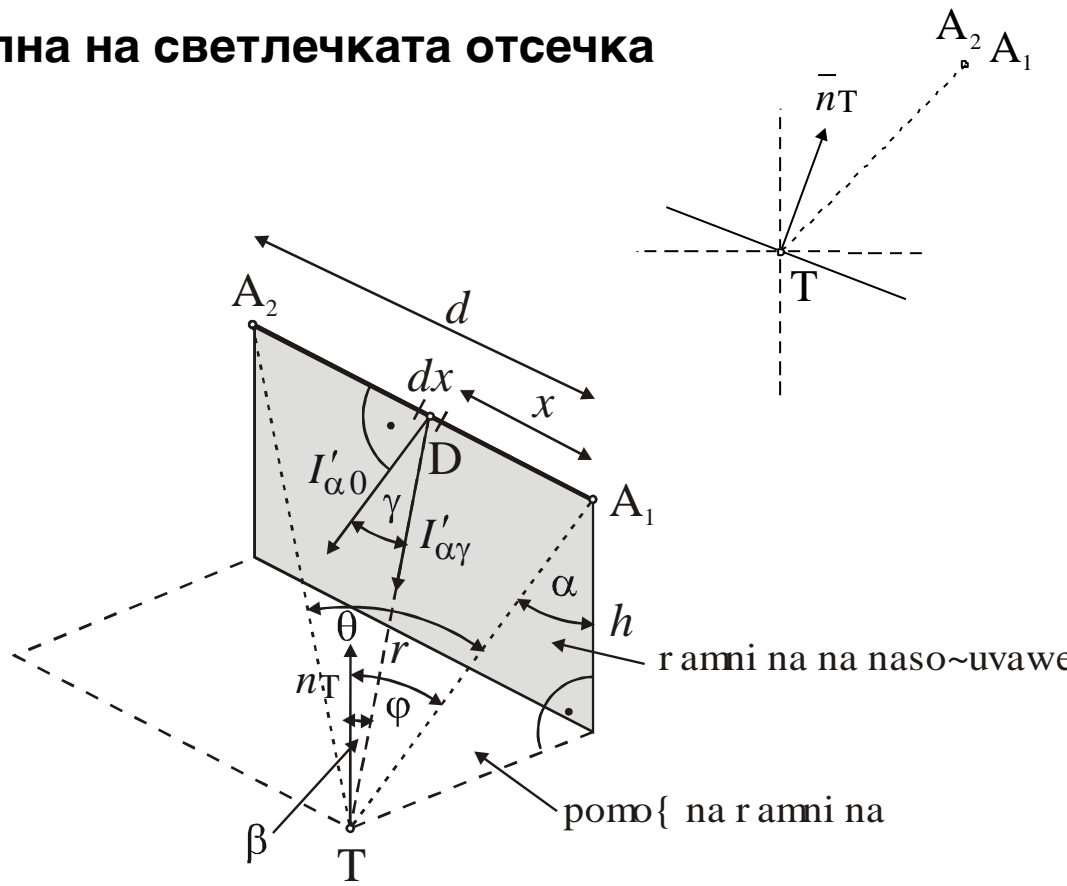
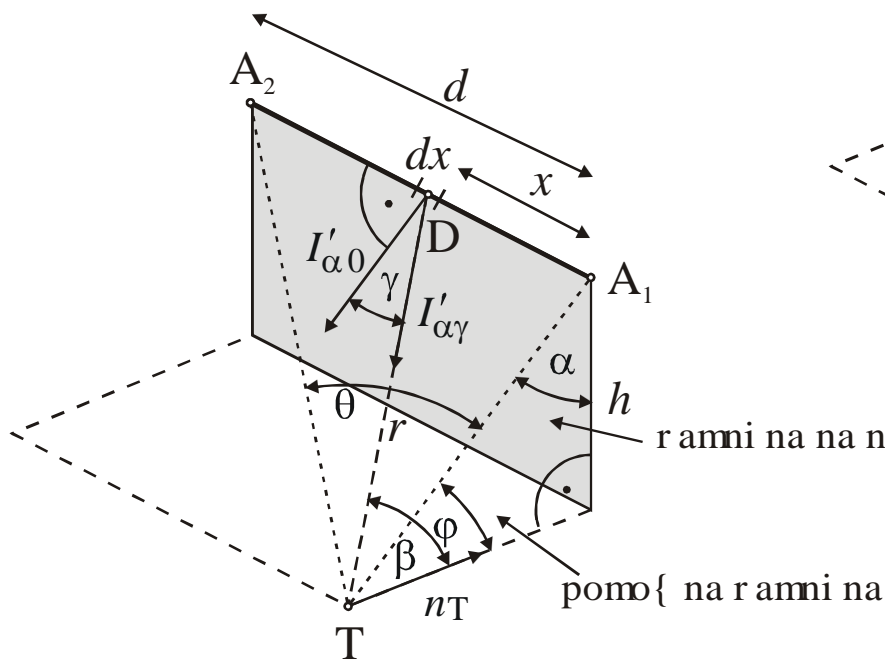
• **Најчести меѓусебни положби на осветлуваната рамнина и светлечката отсечка:**

- осветлуваната рамнина е паралелна на светлечката отсечка
- осветлуваната рамнина е нормална на светлечката отсечка

**Осветлуваната рамнина е паралелна на светлечката отсечка**

$$\cos \beta = \cos \gamma \cdot \cos \varphi.$$

$$E = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^{\theta} \cos^{s+1} \gamma \cdot d\gamma$$



# Осветлуваната рамнина е нормална на светлечката отсечка

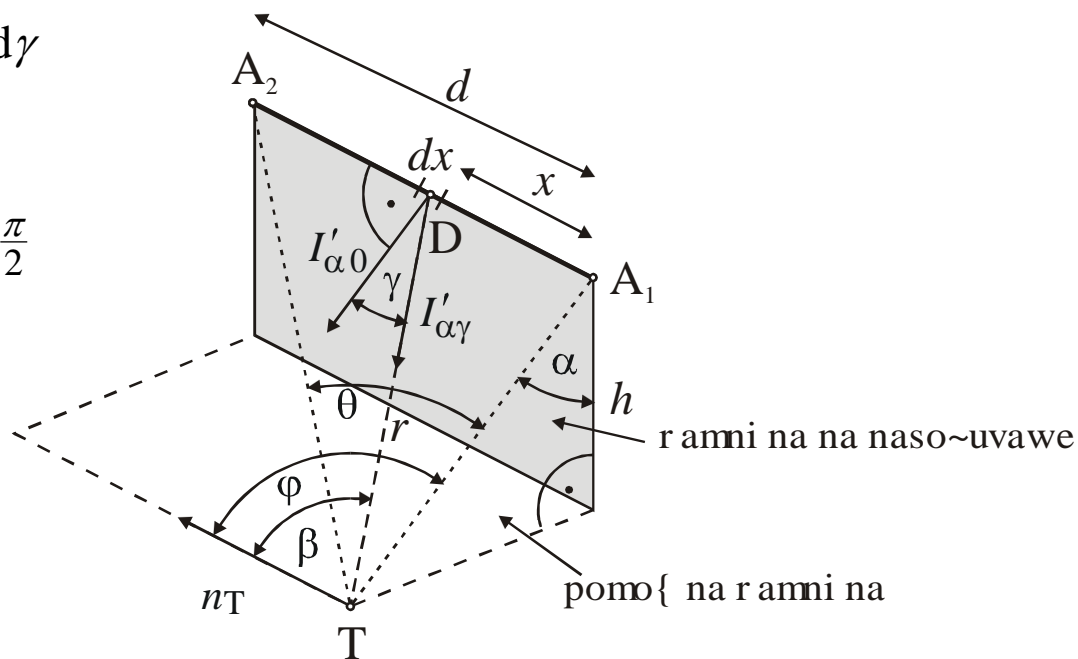
$$E = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \int_0^{\theta} \cos \beta \cdot \cos^s \gamma \cdot d\gamma$$

$$\beta + \gamma = \frac{\pi}{2}$$

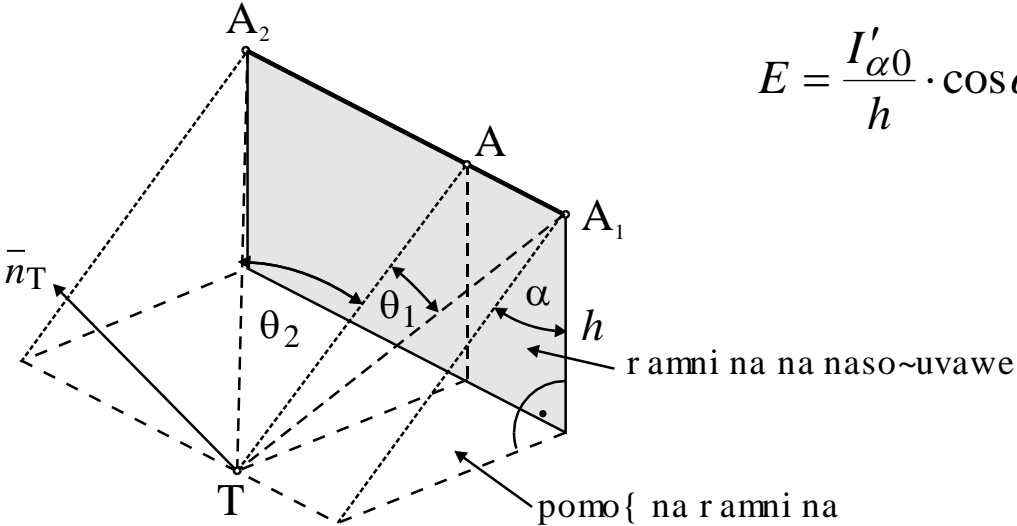
$$\cos \beta = \sin \gamma$$

$$dE = \frac{I'_{\alpha \gamma}}{h} \cdot \cos \alpha \cdot \sin \gamma \cdot d\gamma$$

$$\varphi = \frac{\pi}{2}$$



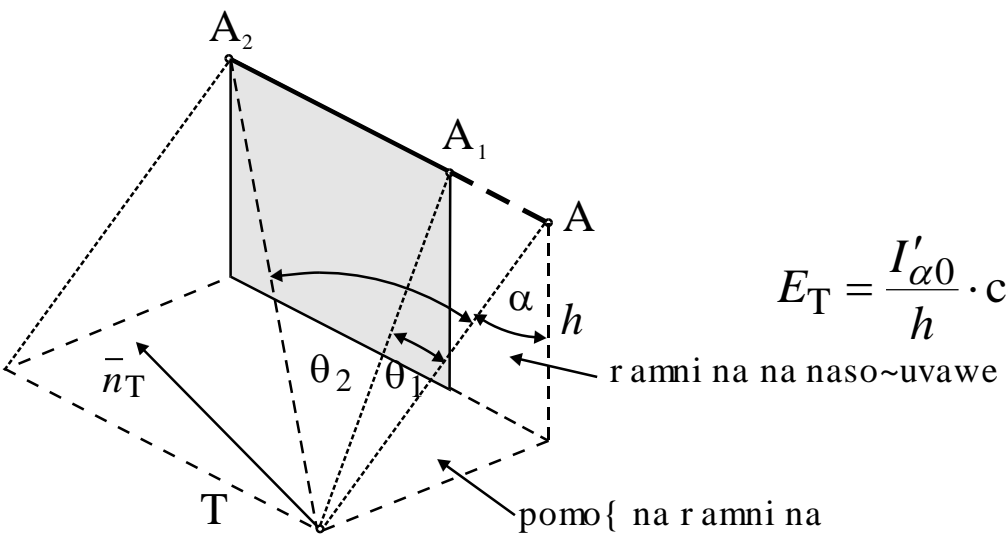
$$E = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \int_0^{\theta} \sin \gamma \cdot \cos^s \gamma \cdot d\gamma$$



$$E = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \int_0^{\theta} \cos \beta \cdot \cos^s \gamma \cdot d\gamma$$

$$E_T = E_{T(\overline{AA_1})} + E_{T(\overline{AA_2})}$$

$$E_T = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \left( \int_0^{\theta_1} \cos \beta \cdot \cos^s \gamma \cdot d\gamma + \int_0^{\theta_2} \cos \beta \cdot \cos^s \gamma \cdot d\gamma \right)$$



$$E_T = E_{T(\overline{AA_2})} - E_{T(\overline{AA_1})}$$

$$E_T = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \left( \int_0^{\theta_2} \cos \beta \cdot \cos^s \gamma \cdot d\gamma - \int_0^{\theta_1} \cos \beta \cdot \cos^s \gamma \cdot d\gamma \right)$$

$$E_T = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \left( \int_{\theta_1}^{\theta_2} \cos \beta \cdot \cos^s \gamma \cdot d\gamma \right)$$

Задача 4.5.3 Коцка е поставена така што нејзините две страни се хоризонтални. Должината на работ на коцката е  $a=3\text{ m}$ . Четири еднакви светлечки отсечки се поставени така што се поклопуваат со рабовите на горната страна на коцката. За секој од изворите е познато дека во секоја рамнина што го содржи изворот максималната светлинска јачина изнесува  $180\text{ cd/m}$  и дека сјајноста на изворот е еднаква во сите правци.

- Да се пресмета хоризонталната осветленост во темињата на коцката што лежат на долната страна на коцката (точки В).
- Да се пресмета хоризонталната осветленост во средиштето на коцката (точка Т).

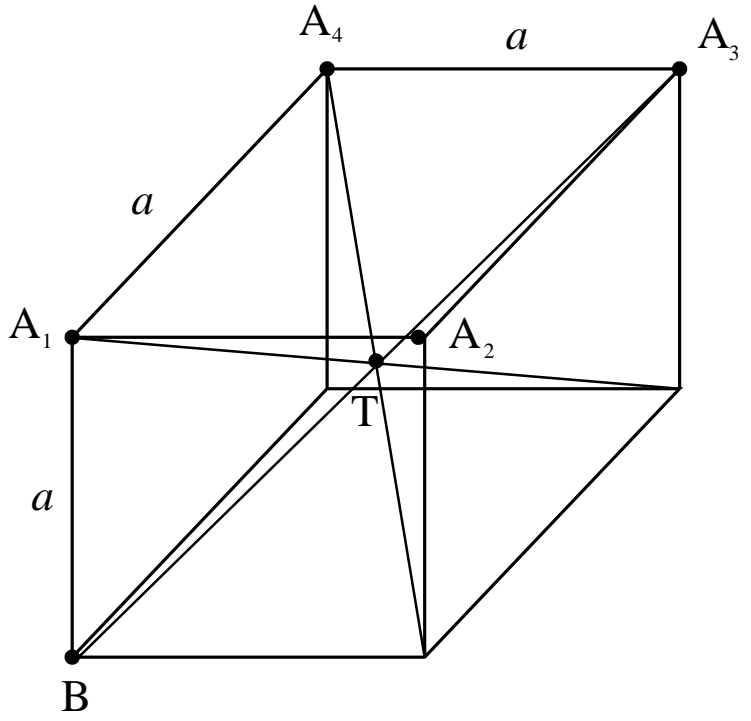
$a = 3\text{ m}; I'_{\alpha 0} = 180\text{ cd/m}$

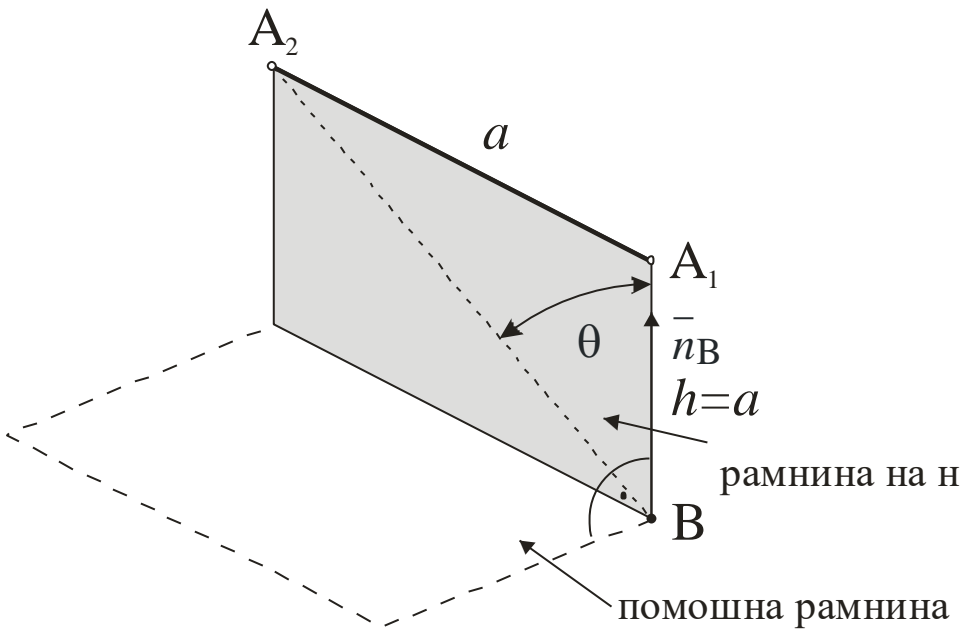
a)  $E_B = ?$  b)  $E_T = ?$  c)  $E_{T_{\text{vert.}} \cdot A_1A_2} = ?$

$I'_{\alpha\gamma} = I'_{\alpha 0} \cdot \cos^s \gamma = I'_{\alpha 0} \cdot \cos \gamma$

$E_B = E_{B_{A_1A_2}} + E_{B_{A_1A_4}} + E_{B_{A_2A_3}} + E_{B_{A_3A_4}}$

$E_B = 2 \cdot E_{B_{A_1A_2}} + 2 \cdot E_{B_{A_3A_4}}$

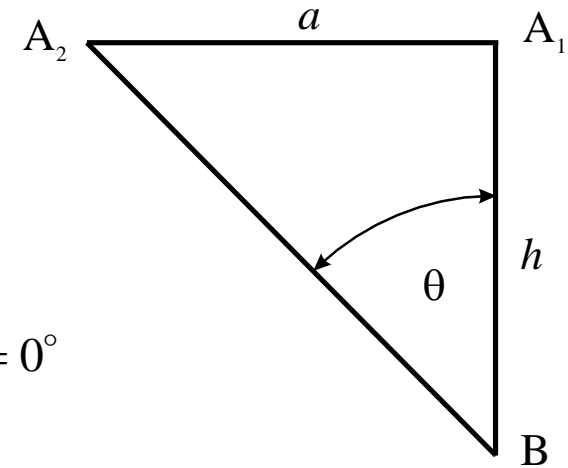




рамнина на насочување  $\alpha = \varphi = 0^\circ$

$$h = a$$

$$\theta = \arctan \frac{a}{h} = \frac{\pi}{4}$$

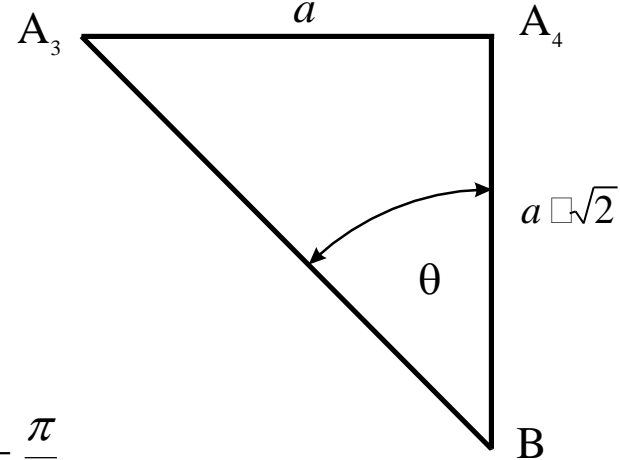


$$E_{B_{A_1A_2}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^\theta \cos^{s+1} \gamma \cdot d\gamma$$

$$E_{B_{A_1A_2}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^\theta \cos^2 \gamma \cdot d\gamma = \frac{180}{3} \cdot \cos 0 \cdot \cos 0 \cdot \int_0^\theta \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{B_{A_1A_2}} = 60 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_0^\theta = 60 \cdot \left( \frac{\pi}{8} + \frac{\sin \frac{\pi}{2}}{4} \right) = 38,6 \text{ lx}$$

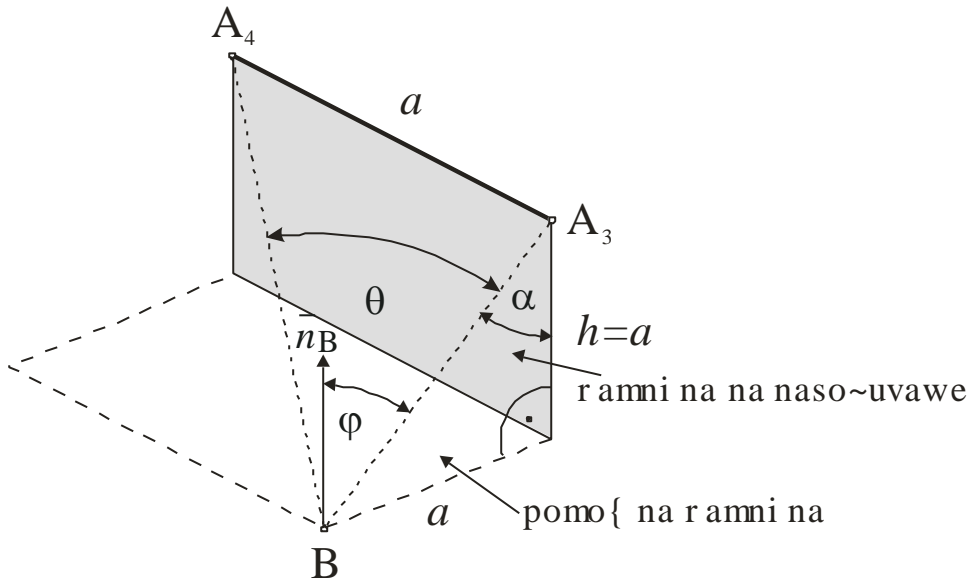
$$E_{B_{A_3A_4}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^{\theta} \cos^{s+1} \gamma \cdot d\gamma$$



$$\alpha = \varphi = \frac{\pi}{4}$$

$$h = a$$

$$\theta = \arctan \frac{a}{a \cdot \sqrt{2}} = 0,6155 \text{ rad}$$



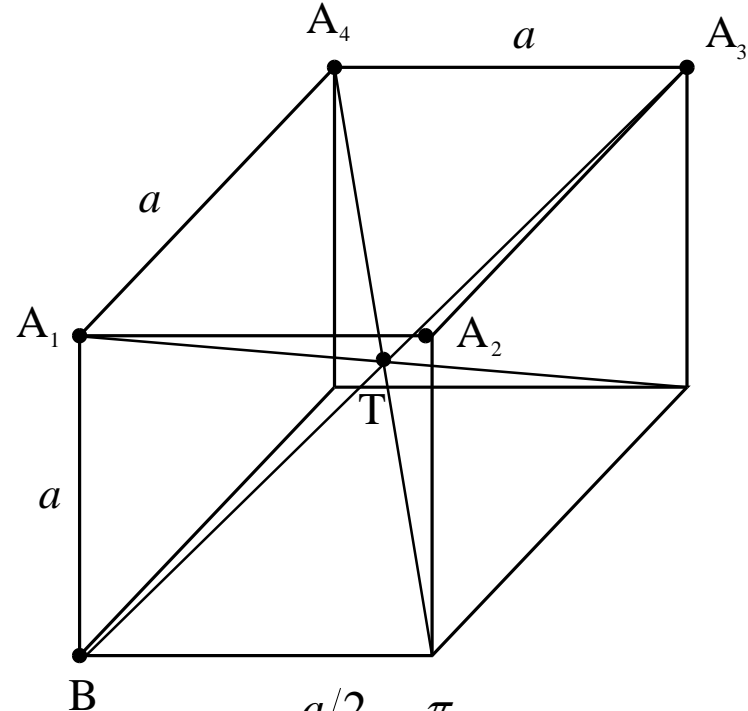
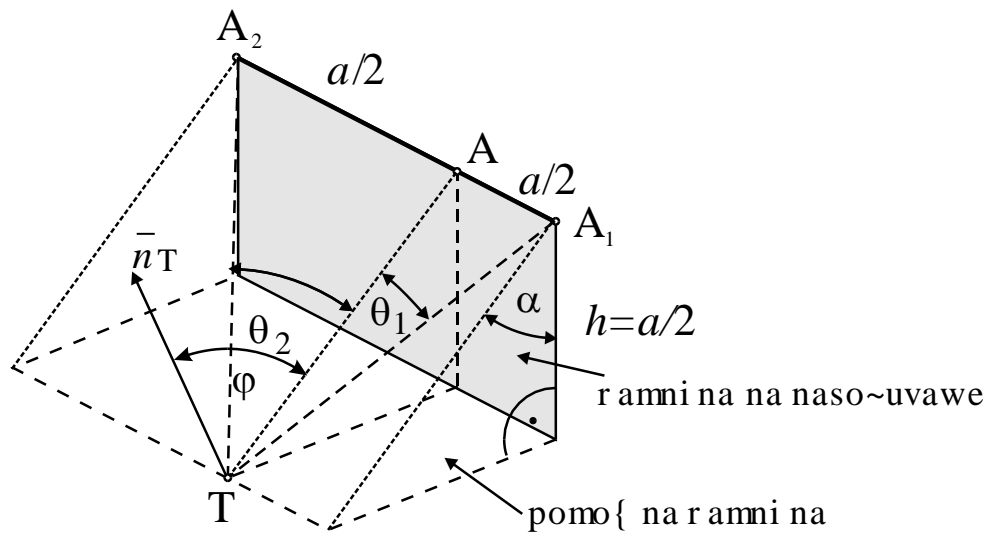
$$E_{B_{A_3A_4}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^{\theta} \cos^2 \gamma \cdot d\gamma = \frac{180}{3} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \cdot \int_0^{\theta} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{B_{A_3A_4}} = 30 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_0^{\theta} = 30 \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 16,3 \text{ lx}$$

$$E_B = 2 \cdot E_{B_{A_1A_2}} + 2 \cdot E_{B_{A_3A_4}} = 2 \cdot 38,6 + 2 \cdot 16,3 = 109,8 \text{ lx}$$

$$E_T = E_{T_{A_1A_2}} + E_{T_{A_1A_4}} + E_{T_{A_2A_4}} + E_{T_{A_3A_4}} = 4 \cdot E_{T_{A_1A_2}}$$

$$E_T = 4 \cdot (E_{T_{AA_1}} + E_{T_{AA_2}}) = 8 \cdot E_{T_{AA_1}}$$



$$\alpha = \varphi = \arctan \frac{a/2}{h} = \frac{\pi}{4}$$

$$h = \frac{a}{2}$$

$$\theta_1 = \theta_2 = \arctan \frac{a/2}{a \cdot \sqrt{2}/2} = 0,6155 \text{ rad}$$

$$E_{T_{AA_1}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^{\theta} \cos^2 \gamma \cdot d\gamma = \frac{180}{1,5} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \cdot \int_0^{\theta_1} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{T_{AA_1}} = 60 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_0^{\theta_1} = 60 \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 32,6 \text{ lx}$$

$$E_T = 8 \cdot E_{T_{AA_1}} = 8 \cdot 32,6 \approx 261 \text{ lx}$$



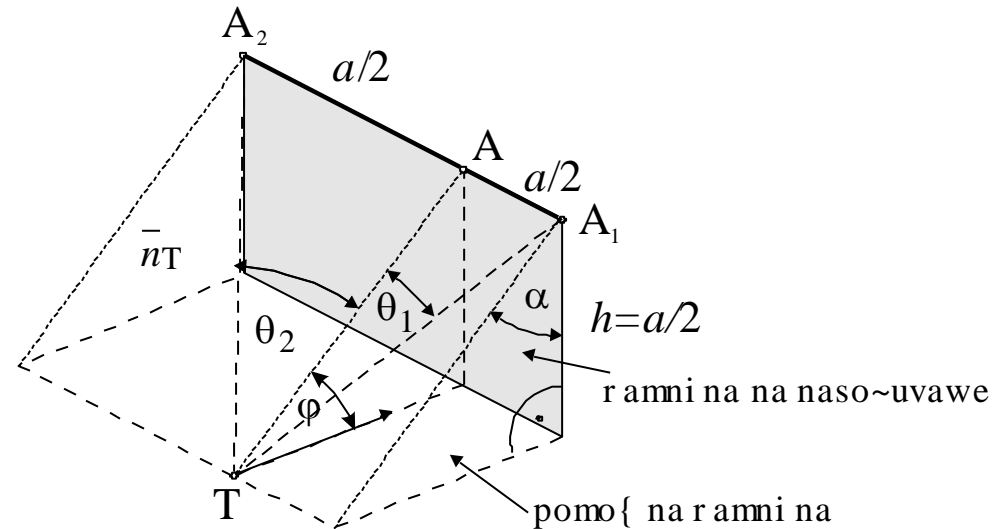
c) вертикална осветленост во T ако вертикалната рамнина е паралелна на отсечката и точката е осветлена само од две отсечки паралелни на неа

$$E_{T_{\text{vert.}} \cdot A_1A_2} = 2 \cdot E_{T_{\text{vert.}} \cdot AA_2}$$

$$\alpha = \varphi = \arctan \frac{a/2}{h} = \frac{\pi}{4}$$

$$h = \frac{a}{2}$$

$$\theta_1 = \theta_2 = \arctan \frac{a/2}{a \cdot \sqrt{2}/2} = 0,6155 \text{ rad}$$



$$E_{T_{\text{vert.}} \cdot A_1A_2} = 2 \cdot E_{T_{\text{vert.}} \cdot AA_2} = 2 \cdot \frac{180}{1,5} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \cdot \int_0^{\theta_1} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{T_{\text{vert.}} \cdot A_1A_2} = 120 \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 65,2 \text{ lx}$$

Напомена: Бидејќи максималната светлинска јачина во сите рамнини што ја содржат светлечката отсечка е еднаква, изборот на референтната рамнина (рамнината на насочување) е произволен. Во сите три случаја може да се избере рамнината на насочување да поминува низ точките во коишто пресметуваме осветленост:

a)

$$\alpha = \varphi = 0^\circ \quad h = a \quad \theta = \arctan \frac{a}{h} = \frac{\pi}{4}$$

$$E_{B_{A_1A_2}} = 60 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_0^\theta = 60 \cdot \left( \frac{\pi}{8} + \frac{\sin \frac{\pi}{2}}{4} \right) = 38,6 \text{ lx}$$

$$\alpha = 0 \quad \varphi = \frac{\pi}{4} \quad h = a \cdot \sqrt{2} \quad \theta = \arctan \frac{a}{h} = \arctan \frac{1}{\sqrt{2}} = 0,6155$$

$$E_{B_{A_3A_4}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^\theta \cos^2 \gamma \cdot d\gamma = \frac{180}{3 \cdot \sqrt{2}} \cdot \cos 0 \cdot \cos \frac{\pi}{4} \cdot \int_0^\theta \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{B_{A_3A_4}} = \frac{180}{3 \cdot \sqrt{2}} \cdot \cos 0 \cdot \cos \frac{\pi}{4} \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 30 \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 16,3 \text{ lx}$$

$$E_B = 2 \cdot E_{B_{A_1A_2}} + 2 \cdot E_{B_{A_3A_4}} = 2 \cdot 38,6 + 2 \cdot 16,3 = 109,8 \text{ lx}$$

$$b) \quad h = \frac{a \cdot \sqrt{2}}{2} \quad \alpha = 0 \quad \varphi = \arccos \frac{a/2}{h} = \frac{\pi}{4} \quad \theta_1 = \theta_2 = \arctan \frac{a/2}{a \cdot \sqrt{2}/2} = 0,6155 \text{ rad}$$

$$E_T = 4 \cdot (E_{T_{AA_1}} + E_{T_{AA_2}}) = 8 \cdot E_{T_{AA_1}}$$

$$E_{T_{AA_1}} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_0^{\theta} \cos^2 \gamma \cdot d\gamma = \frac{180}{1,5 \cdot \sqrt{2}} \cdot 1 \cdot \frac{\sqrt{2}}{2} \cdot \int_0^{\theta_1} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{T_{AA_1}} = 60 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_0^{\theta_1} = 60 \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 32,6 \text{ lx}$$

$$c) \quad h = \frac{a}{2} \cdot \sqrt{2} \quad \alpha = 0 \quad \varphi = \arccos \frac{a/2}{h} = \frac{\pi}{4} \quad \theta_1 = \theta_2 = \arctan \frac{a/2}{a \cdot \sqrt{2}/2} = 0,6155 \text{ rad}$$

$$E_{T_{\text{vert.} \cdot A_1 A_2}} = 2 \cdot E_{T_{\text{vert.} \cdot AA_2}}$$

$$E_{T_{\text{vert.} \cdot A_1 A_2}} = 2 \cdot E_{T_{\text{vert.} \cdot AA_2}} = 2 \cdot \frac{180}{1,5 \cdot \sqrt{2}} \cdot 1 \cdot \frac{\sqrt{2}}{2} \cdot \int_0^{\theta_1} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{T_{\text{vert.} \cdot A_1 A_2}} = 120 \cdot \left( \frac{0,6155}{2} + \frac{\sin 1,2310}{4} \right) = 65,2 \text{ lx}$$

### Задача 4.5.8

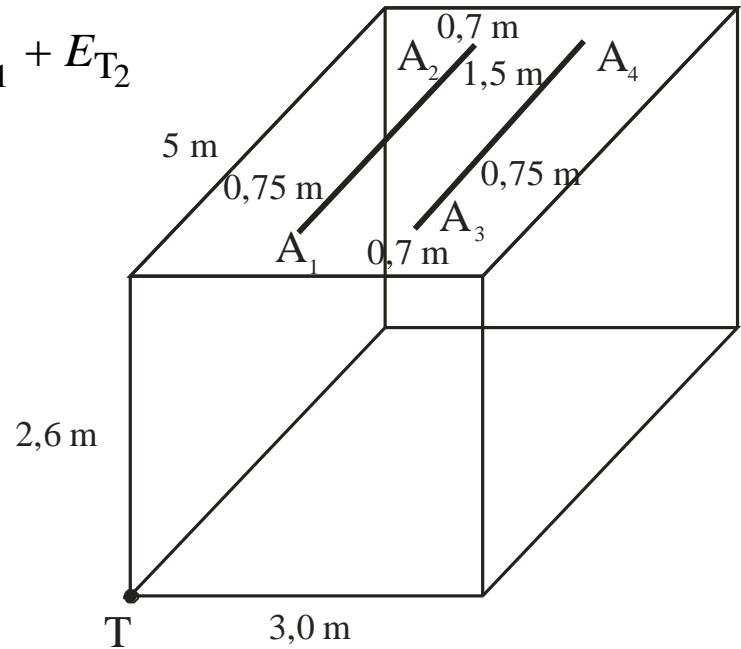
$$\Phi_{si.j.} = 3200 \text{ lm}$$

$$\Phi = \pi^2 \cdot I_{\max} = \pi^2 \cdot I_{\alpha 0}$$

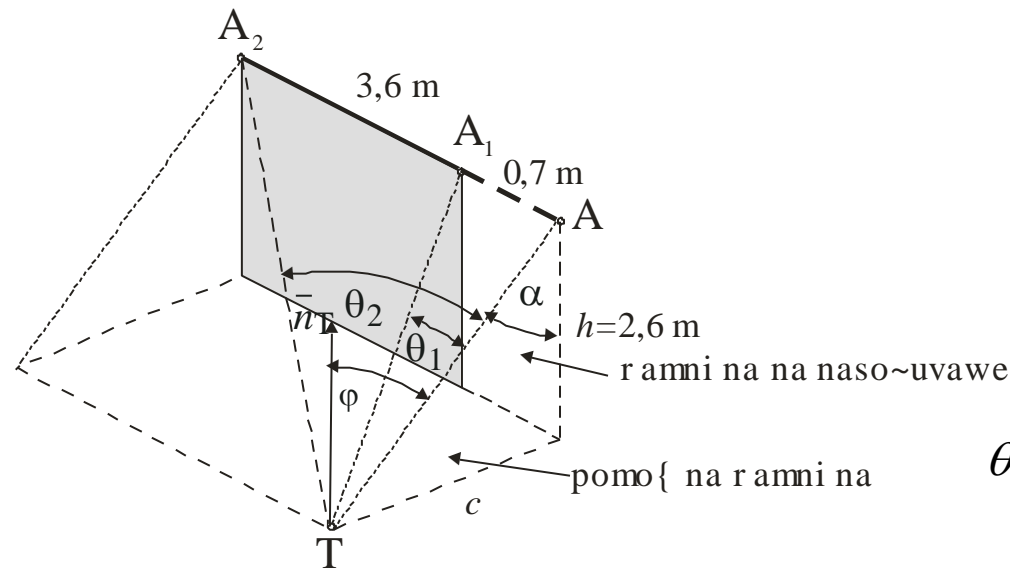
$$I_{\alpha 0} = \frac{\Phi}{\pi^2} = \frac{3200}{\pi^2} = 324,23 \text{ cd}$$

$$I'_{\alpha 0} = \frac{I_{\alpha 0}}{1,2} = \frac{324,23}{1,2} \approx 270 \text{ cd/m}$$

$$E_T = E_{T_{A_1 A_2}} + E_{T_{A_3 A_4}} = E_{T_1} + E_{T_2}$$



### Осветленост од отсечката A<sub>1</sub>A<sub>2</sub>



$$\overline{AT} = \sqrt{h^2 + c^2} = \sqrt{2,6^2 + 0,75^2} \approx 2,7 \text{ m}$$

$$\alpha = \varphi = \arctan \frac{c}{h} = \arctan \frac{0,75}{2,6} = 0,2808 \text{ rad}$$

$$\theta_1 = \arctan \frac{0,7}{\overline{AT}} = \arctan \frac{0,7}{2,7} = 0,2537 \text{ rad}$$

$$\theta_2 = \arctan \frac{0,7 + 3,6}{\overline{AT}} = \arctan \frac{4,3}{2,7} = 1,0101 \text{ rad}$$

$$E_{T_1} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_{\theta_1}^{\theta_2} \cos^2 \gamma \cdot d\gamma = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_{\theta_1}^{\theta_2} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{T_1} = \frac{270}{2,6} \cdot \cos^2 0,2808 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_{\theta_1}^{\theta_2}$$

$$E_{T_1} = \frac{270}{2,6} \cdot \cos^2 0,2808 \cdot \left( \frac{\theta_2 - \theta_1}{2} + \frac{\sin 2\theta_2 - \sin 2\theta_1}{4} \right) \quad E_{T_1} = \frac{270}{2,6} \cdot \cos^2 0,2808 \cdot 0,4819 = 46,2 \text{ lx}$$

Осветленост од отсечката  $A_3A_4$

$$\overline{AT} = \sqrt{h^2 + c^2} = \sqrt{2,6^2 + 2,25^2} \approx 3,4 \text{ m} \quad \alpha = \varphi = \arctan \frac{c}{h} = \arctan \frac{2,25}{2,6} = 0,7134 \text{ rad}$$

$$\theta_1 = \arctan \frac{0,7}{\overline{AT}} = \arctan \frac{0,7}{3,4} = 0,2030 \text{ rad} \quad \theta_2 = \arctan \frac{0,7 + 3,6}{\overline{AT}} = \arctan \frac{4,3}{3,4} = 0,9018 \text{ rad}$$

$$E_{T_2} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_{\theta_1}^{\theta_2} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma = \frac{270}{2,6} \cdot \cos^2 0,7134 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_{\theta_1}^{\theta_2}$$

$$E_{T_2} = 47,14 \cdot \left( \frac{\theta_2 - \theta_1}{2} + \frac{\sin 2\theta_2 - \sin 2\theta_1}{4} \right) \quad E_{T_2} = 47,14 \cdot 0,4939 = 23,3 \text{ lx}$$

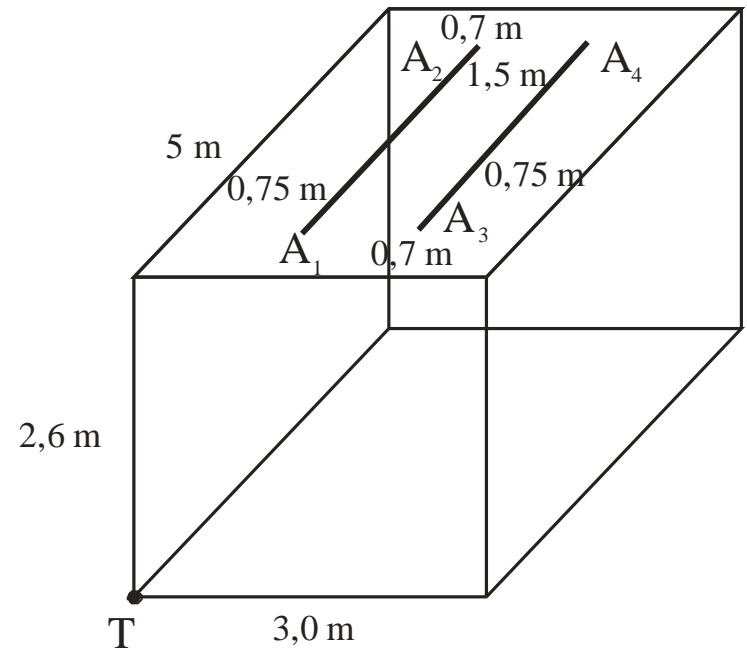
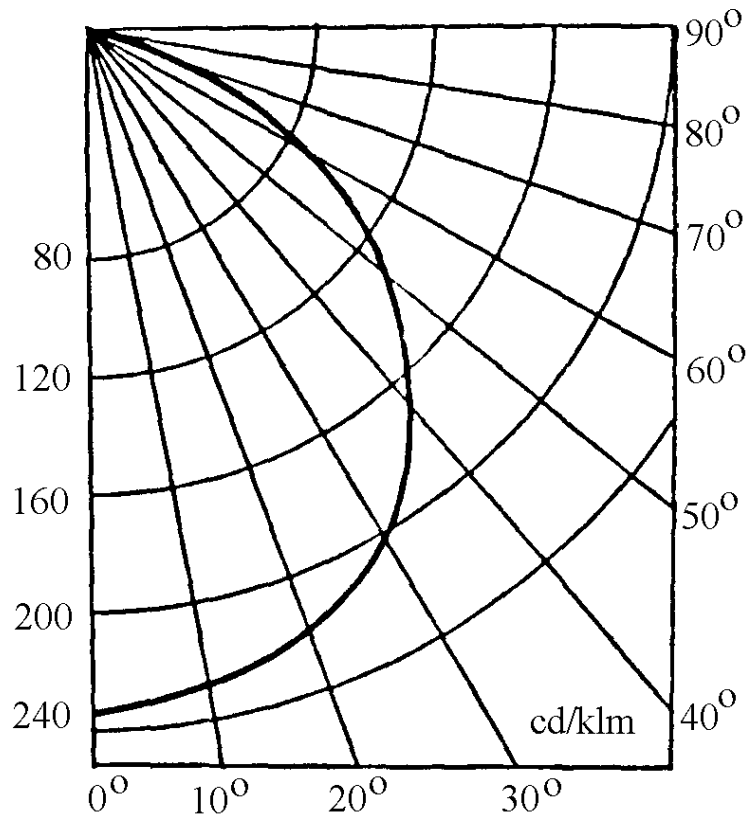
$$E_T = E_{T_1} + E_{T_2} = 46,2 + 23,3 = 69,5 \text{ lx}$$

# Задача 4.5.9

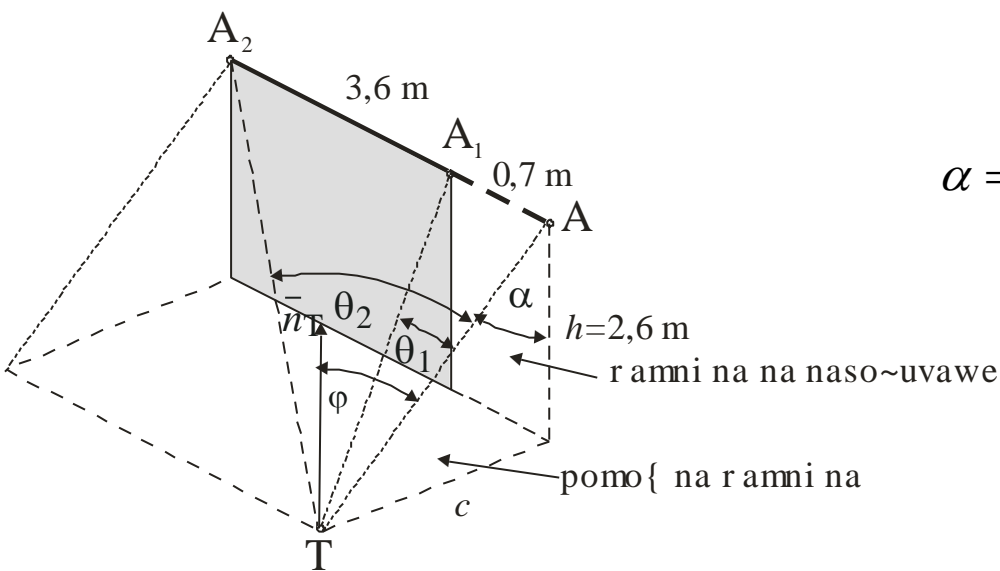
$$E_T = E_{T_{A_1A_2}} + E_{T_{A_3A_4}} = E_{T_1} + E_{T_2}$$

$$\Phi_{si j} = 3200 \text{ lm}$$

$$I'_{\alpha 0} = \frac{I_{\alpha 0}}{1,2} = \frac{3,2 \cdot I_{\alpha}(1000 \text{ lm})}{1,2}$$



# Осветленост од отсечката $A_1A_2$



$$\overline{AT} = \sqrt{h^2 + c^2} = \sqrt{2,6^2 + 0,75^2} \approx 2,7 \text{ m}$$

$$\alpha = \varphi = \arctan \frac{c}{h} = \arctan \frac{0,75}{2,6} = 0,2808 \text{ rad} = 16^\circ$$

$$\theta_1 = \arctan \frac{0,7}{\overline{AT}} = \arctan \frac{0,7}{2,7} = 0,2537 \text{ rad}$$

$$\theta_2 = \arctan \frac{0,7 + 3,6}{\overline{AT}} = \arctan \frac{4,3}{2,7} = 1,0101 \text{ rad}$$

$$I'_{16;0} = \frac{I_{16;0}}{1,2} = \frac{3,2 \cdot 225}{1,2} = 600 \text{ cd/m}$$

$$E_{T_1} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_{\theta_1}^{\theta_2} \cos^2 \gamma \cdot d\gamma = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_{\theta_1}^{\theta_2} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma$$

$$E_{T_1} = \frac{600}{2,6} \cdot \cos^2 0,2808 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_{\theta_1}^{\theta_2}$$

$$E_{T_1} = \frac{600}{2,6} \cdot \cos^2 0,2808 \cdot \left( \frac{\theta_2 - \theta_1}{2} + \frac{\sin 2\theta_2 - \sin 2\theta_1}{4} \right) \quad E_{T_1} = \frac{600}{2,6} \cdot \cos^2 0,2808 \cdot 0,4819 = 102,2 \text{ lx}$$

## Осветленост од отсечката $A_3A_4$

$$\overline{AT} = \sqrt{h^2 + c^2} = \sqrt{2,6^2 + 2,25^2} \approx 3,4 \text{ m} \quad \alpha = \varphi = \arctan \frac{c}{h} = \arctan \frac{2,25}{2,6} = 0,7134 \text{ rad} \approx 40^\circ$$

$$\theta_1 = \arctan \frac{0,7}{\overline{AT}} = \arctan \frac{0,7}{3,4} = 0,2030 \text{ rad} \quad \theta_2 = \arctan \frac{0,7 + 3,6}{\overline{AT}} = \arctan \frac{4,3}{3,4} = 0,9018 \text{ rad}$$

$$I'_{40;0} = \frac{I_{40;0}}{1,2} = \frac{3,2 \cdot 170}{1,2} = 453 \text{ cd/m}$$

$$E_{T_2} = \frac{I'_{\alpha 0}}{h} \cdot \cos \alpha \cdot \cos \varphi \cdot \int_{\theta_1}^{\theta_2} \frac{1 + \cos 2\gamma}{2} \cdot d\gamma = \frac{453}{2,6} \cdot \cos^2 0,7134 \cdot \left( \frac{\gamma}{2} + \frac{\sin 2\gamma}{4} \right) \Big|_{\theta_1}^{\theta_2}$$

$$E_{T_2} = 99,6 \cdot \left( \frac{\theta_2 - \theta_1}{2} + \frac{\sin 2\theta_2 - \sin 2\theta_1}{4} \right)$$

$$E_{T_2} = 99,6 \cdot 0,4939 = 49,2 \text{ lx}$$

$$E_T = E_{T_1} + E_{T_2} = 102,2 + 49,2 = 151,4 \text{ lx}$$